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Corrigendum

Corrigendum to "Rescaled variance and related tests for long memory in volatility and levels" [J. Econom. 112 (2003) 265–294]

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A minor correction is needed in the proof of Theorem 3.1.

Theorem 3.1 shows that in the case of a long memory time series $\{X_k\}$ with autocovariances $\gamma_j = \mathrm{Cov}(X_j, X_0) \sim cj^{2d-1}$, $0 < d < \frac{1}{2}$, the estimate of spectral density at zero frequency $\hat{s}^2_{N,q} = \hat{\gamma}_0 + 2\sum_{j=1}^{q}(1 - j/(q+1))\hat{\gamma}_j$ can be used to estimate the asymptotic variance of the renormalised sample mean:

$$q^{-2d}\hat{s}^2_{N,q} \xrightarrow{\mathrm{P}} c_d^2 = \lim_{N\to\infty} \mathrm{Var}\left(N^{-1/2-d}\sum_{j=1}^{N} X_j\right).$$

The convergence (A.1) on p. 290 is correct but two corrections on p. 291 must be done. It is obvious that line 5, p. 291 should be $\mathrm{E}(v_{N,1} - \mathrm{E}v_{N,2})^2 = \mathrm{o}(q^{4d})$. Since $i_{N,1} = \mathrm{o}(q^{2d}) = \mathrm{o}(q^{4d})$ we have to show that $i_{N,2} = \mathrm{o}(q^{4d})$. To prove that we must correct the last line on p. 291. Note that summation is taken over $|j|, |j'| \leqslant q$ and

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$|i|, |i'| \leqslant N$. We have to apply here the estimate

$$\sum_{i=1}^{k} |\gamma_{i+v}| \leqslant Ck^{2d} \tag{1}$$

uniformly in $v = 0, \pm 1, \pm 2, \ldots$ and $k \geqslant 1$ which implies that $\sum_{i=1}^{N} |\gamma_{i+v}| \leqslant CN^{2d}$, $\sum_{i=1}^{q} |\gamma_{i+v'}| \leqslant Cq^{2d}$, uniformly in v, v'. Using these bounds in the last line of p. 291 (and correcting the summation) we obtain that $i_{N,2} \leqslant CN^{-2}NqN^{2d}q^{2d} = C(q/N)^{1-2d}q^{4d} = \mathrm{o}(q^{4d})$ to complete the proof.

To prove (1), note that for $|v| \geqslant k+1$ and $1 \leqslant i \leqslant k$, $|\gamma_{i+v}| \leqslant C|i+v|^{2d-1} \leqslant C|k+1-i|^{2d-1}$. Therefore

$$\begin{aligned}\sum_{i=1}^{k} |\gamma_{i+v}| &= \sum_{i=1}^{k} |\gamma_{i+v}| \mathbf{1}_{\{|v| \leqslant k\}} + \sum_{i=1}^{k} |\gamma_{i+v}| \mathbf{1}_{\{|v| \geqslant k+1\}} \\ &\leqslant C\left(\sum_{i=1}^{k} |i+v|^{2d-1} \mathbf{1}_{\{i+v \neq 0,\ |v| \leqslant k\}} + \sum_{i=1}^{k} |k+1-i|^{2d-1} \right) \\ &\leqslant C \sum_{i=1}^{2k} i^{2d-1} \leqslant Ck^{2d}.\end{aligned}$$

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