Interaction Models for Common Long-Range Dependence in Asset Prices Volatility

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Abstract. We consider a class of microeconomic models with interacting agents which replicate the main properties of asset prices time series: non-linearities in levels and common degree of long-memory in the volatilities and co-volatilities of multivariate time series. For these models, long-range dependence in asset price volatility is the consequence of swings in opinions and herding behavior of market participants, which generate switches in the heteroskedastic structure of asset prices. Thus, the observed long-memory in asset prices volatility might be the outcome of a change-point in the conditional variance process, a conclusion supported by a wavelet anaysis of the volatility series. This explains why volatility processes share only the properties of the second moments of long-memory processes, but not the properties of the first moments.

1 Long-Range Dependence in Finance

Asset prices time series are characterized by several features: leptokurtic distribution, nonlinear variations, volatility clustering, unit roots in the conditional mean, and strong dependence in the volatility. These empirical features have been documented in [38,39], [48], [9], [22,23], [3] among others.

Daily prices P_t are modeled by martingale processes, i.e., $E(P_{t+1}|I_t) = P_t$, where I_t denotes the information set available at time t. This property is termed as 'Efficient Market Hypothesis', the content of I_t defining the type of market efficiency considered, see e.g., Fama [13]. As a consequence, the returns $R_t = \log(P_t/P_{t-1})$ are uncorrelated and unpredictable.

However, the power transformation $|R_t|^\delta$ displays strong dependence, the degree of which is the highest for $\delta = 1$. This empirical feature, termed as 'Taylor effect' [48], motivated the use of the class of long-memory volatility models introduced by Robinson [45], and developed in [22], [10], [18] and other works.

This statistical univariate approach was incomplete, as a multivariate analysis, pioneered by Teyssière [50,51], revealed that several time series share a common degree of strong dependence in their conditional variances and covariances. This regularity suggested the presence of a common structural model generating these features.

Furthermore, the series $|R_t|^\delta$ differ from standard long-range dependent, henceforth LRD, processes: while the autocorrelation function and the spectrum of the series $|R_t|^\delta$ display a LRD-type behavior, the series $|R_t|^\delta$ are not trended unlike standard LRD processes. Recent works, see [42], [34], [25], [32],

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considered the change-point problem for volatility processes, as the class of non-homogenous stochastic variance processes is also able to match the empirical properties of asset prices returns.

These empirical results motivated further research for devising structural microeconomic models explaining these features. Kirman and Teyssière [31,29,30] produced a model, based on microeconomic models with interacting agents, which generates these empirical properties of asset prices.

This chapter is organized as follows. Section 2 reviews some statistical methods used for testing for long-range dependence and for the presence of a change-point in the volatility process. Section 3 presents the class of microeconomic models generating the empirical property of common long-range dependence in multivariate asset price volatility. Simulation results for our models are given in Sect. 4.

2 Long-Range Dependent vs. Change-Point Processes

A stationary process Y_t is called a stationary process with long-memory if its autocorrelation function, henceforth ACF, $\rho(k)$ has asymptotically the following hyperbolic rate of decay, see [2], [20], [21], [26], [46]:

$$\rho(k) \sim L(k)k^{2d-1} \quad \text{as } k \to \infty, \tag{1}$$

where $L(k)$ is a slowly varying function,[1] and $d \in (0, 1/2)$ is the long-memory parameter which governs the slow rate of decay of the ACF and then parsimoniously summarizes the degree of long-range dependence of the series. Equivalently, the spectrum $f(\lambda)$ of a long-memory process can be approximated in the neighborhood of the zero frequency as

$$f(\lambda) \sim G\lambda^{-2d}, \quad \text{as } \lambda \to 0^+, \quad 0 < G < \infty. \tag{2}$$

2.1 Statistical Inference

Since the statistical characteristics of volatility processes are more complex than the ones of standard parametric long-memory processes, we resort in this study to semiparametric statistical tools which require mild assumptions on the process generating the data, henceforth DGP.

Several tests for stationarity against long-range dependent alternatives have been proposed by Lo [37], Kwiatkowski et al. [36], and Giraitis, Kokoszka, Leipus and Teyssière [15,16]. These statistics are based on the partial sum process $S_k = \sum_{t=1}^{k}(Y_t - \bar{Y})$ and the assumption that under the null hypothesis of stationarity, the standardized partial sum process satisfies a functional central limit theorem. Lo [37] considered the standardized range of S_k, i.e.,

$$R/S(q) = \frac{1}{\hat{s}_T(q)}\left[\max_{1\le k\le T} S_k - \min_{1\le k\le T} S_k\right] = \frac{\hat{R}_T}{\hat{s}_T(q)}. \tag{3}$$

[1] A function $L(k)$, $k \geq 0$, is called slowly varying function if $L(\lambda k)/L(k) \to 1$ as $k \to \infty$, $\forall \lambda > 0$.

Kwiatkowski et al. [36] considered the standardized second moment of S_k:

$$KPSS(q) = \frac{1}{T^2 \hat{s}_T^2(q)} \sum_{k=1}^{T} S_k^2 = \frac{\hat{M}_T}{T\hat{s}_T^2(q)}, \tag{4}$$

while Giraitis et al. [16] considered the standardized variance of S_k:

$$V/S(q) = \frac{1}{T^2 \hat{s}_T^2(q)} \left[\sum_{k=1}^{T} S_k^2 - \frac{1}{T} \left(\sum_{k=1}^{T} S_k \right)^2 \right] = \frac{\hat{V}_T}{T\hat{s}_T^2(q)}, \tag{5}$$

where $\hat{s}_T^2(q)$ is the heteroskedastic and autocorrelation consistent variance estimator, see [44]:

$$\hat{s}_T^2(q) = T^{-1} \sum_{i=1}^{T} (Y_i - \bar{Y})^2 + 2 \sum_{i=1}^{q} \omega_i(q) \hat{\gamma}_i \quad \text{with} \quad \omega_i(q) = 1 - \frac{1}{q+1}, \tag{6}$$

where the sample auto-covariances $\hat{\gamma}_i$ at lag i account for the possible short-range dependence up to the q^{th} order.

Under the null hypothesis of no long-range dependence, the R/S statistic has the following asymptotic distribution: $T^{-1/2} R/S(q) \xrightarrow{d} \max_{0 \le t \le 1} W^0(t) - \min_{0 \le t \le 1} W^0(t)$, i.e., the range of a Brownian bridge $W^0(t) = W(t) - tW(1)$, on the unit interval, $KPSS(q) \xrightarrow{d} U_{KPSS} = \int_0^1 (W^0(t))^2 dt$, while $V/S(q) \xrightarrow{d} U_{V/S} = \int_0^1 (W^0(t))^2 dt - (\int_0^1 W^0(t) dt)^2$. The V/S statistic is less sensitive to the choice of the truncation order q than the R/S statistic, and is more powerful than the KPSS statistic. Furthermore, $E(U_{KPSS}) = 1/6$, $V(U_{KPSS}) = 1/45$, while $E(U_{V/S}) = 1/12$ and $V(U_{V/S}) = 1/360$. The smaller variance of the random variable V/S might explain its superior power for small samples.

The R/S statistic has been used by Mandelbrot and his co-authors, see [40], for estimating the degree of long-range dependence d. Define $\hat{s}_T^2 = \hat{s}_T^2(0)$, then $\hat{s}_T^2 \to \mathrm{Var}(Y)$. Since

$$S_k = \sum_{j=1}^{k} (Y_j - EY_j) - \frac{k}{T} \sum_{j=1}^{T} (Y_j - EY_j), \tag{7}$$

and

$$\frac{1}{T^{1/2+d}} \sum_{j=1}^{[Tt]} (Y_j - EY_j) \xrightarrow{D[0,1]} C\, W_{1/2+d}(t), \tag{8}$$

where C is a positive constant, and $\xrightarrow{D[0,1]}$ means weak convergence in the space $D[0,1]$ endowed with Skorokhod topology. Then

$$\frac{\hat{R}_T}{T^{1/2+d}} \xrightarrow{d} C \left[\max_{0 \le t \le 1} W^0_{1/2+d}(t) - \min_{0 \le t \le 1} W^0_{1/2+d}(t) \right], \tag{9}$$

$W^0_{1/2+d}(t)$ being the fractional Brownian bridge, defined as

$$W^0_{1/2+d}(t) = W_{1/2+d}(t) - tW_{1/2+d}(1). \tag{10}$$

Thus,

$$\frac{1}{T^{1/2+d}}\frac{\hat{R}_T}{\hat{s}_T} \xrightarrow{d} \frac{C\left[\max_{0\leq t\leq 1} W^0_{1/2+d}(t) - \min_{0\leq t\leq 1} W^0_{1/2+d}(t)\right]}{\mathrm{Var}(Y)^{1/2}}, \tag{11}$$

Equation (11) constitutes a theoretical foundation for the R/S estimator. Taking logarithms of both sides yields the heuristic identity:

$$\log\left(\hat{R}_T/\hat{s}_T\right) \approx \left(\frac{1}{2}+d\right)\log T + \mathbf{constant}, \quad \text{as } T \to \infty, \tag{12}$$

Denote $\hat{d}_{R/S} = (\log(\hat{R}_T/\hat{s}_T)/\log T) - 1/2$, then $\hat{d}_{R/S} - d = O_P(1/\log T)$. Thus, $1/2 + d$ can be interpreted as the slope of a regression line of $\log(\hat{R}_T/\hat{s}_T)$ on $\log T$.

Giraitis, Kokoszka, Leipus and Teyssière [17] suggested to extend this principle to the KPSS and the V/S statistics. By (8)

$$\frac{\hat{M}_T}{T^{1+2d}} \xrightarrow{d} C^2 \int_0^1 \left[W^0_{1/2+d}(t)\right]^2 dt. \tag{13}$$

Define $\hat{d}_{KPSS} = (\log(\hat{M}_T^{1/2}/\hat{s}_T)/\log T) - 1/2$, we get $\hat{d}_{KPSS} - d = O_P(1/\log T)$. Thus, the slope of the regression line of $\log(\hat{M}_T^{1/2}/\hat{s}_T)$ on $\log T$ estimates $d+1/2$. Similarly, the regression of $\log(\hat{V}_T^{1/2}/\hat{s}_T)$ on $\log T$ estimates $d + 1/2$. Setting $\hat{d}_{V/S} = (\log(\hat{V}_T^{1/2}/\hat{s}_T)/\log T) - 1/2$, we get $\hat{d}_{V/S} - d = O_P(1/\log T)$.

The technical details of the implementation of these 'pox-plot' estimators are described in [2] and [17]. These semiparametric estimators have a few drawbacks. There is no formal asymptotic theory for them, and they have the slow rate of convergence of order $\log(T)$. For that reason, we complete the empirical study of the long-range dependent properties of our microeconomic model by considering another semi-parametric estimator of the degree of long-range dependence proposed by Robinson [47], which is the discrete version of the Whittle approximate maximum likelihood estimator in the spectral domain. This estimator, suggested by Kunsch [35], is based on the mild assumption (2) of the spectrum $f(\lambda)$ of a long-memory process in the neighborhood of the zero frequency. The consequences of a misspecification of the functional form of the spectrum in the Whittle estimator are avoided with this local approximation. After concentrating in G, the estimator is given by:

$$\hat{d} = \arg\min_d \left\{ \ln\left(\frac{1}{m}\sum_{j=1}^m \frac{I(\lambda_j)}{\lambda_j^{-2d}}\right) - \frac{2d}{m}\sum_{j=1}^m \ln(\lambda_j) \right\}, \tag{14}$$

where $I(\lambda_j)$ is the periodogram estimated for the range of Fourier frequencies $\lambda_j = \pi j/T, j = 1, \ldots, m \ll [T/2]$, the bandwidth parameter m tends to infinity with T, but more slowly since $1/m + m/T \to 0$ as $T \to \infty$. Under appropriate conditions, which include the existence of a moving average representation and the differentiability of the spectrum near the zero frequency, this estimator has the following distribution independent of the value of d:

$$\sqrt{m}(\hat{d} - d) \sim N(0, 1/4) . \tag{15}$$

Furthermore, this estimator is robust to the presence of conditional heteroskedasticity of general form, and an optimal bandwidth with the same robustness properties does exist under mild assumptions, see [24].

2.2 Long-Memory Volatility Models

The clustering of the variations of asset returns can be modeled by the class of Generalized Autoregressive Conditional Heteroskedastic GARCH(1,1), processes, see [7] and [48], defined as:

$$R_t = \mu + \varepsilon_t, \quad \sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2, \quad \varepsilon_t \sim N(0, \sigma_t^2), \tag{16}$$

with $\omega > 0$, and $\alpha,\ \ \beta \geq 0$. It has been empirically found that for large samples the sum of the estimated parameters $\hat{\alpha} + \hat{\beta}$ was close to one, the restricted model being an Integrated GARCH(1,1), henceforth IGARCH(1,1) see [11], defined as:

$$R_t = \mu + \varepsilon_t, \quad \sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + (1 - \beta)\varepsilon_{t-1}^2, \quad \varepsilon_t \sim N(0, \sigma_t^2), \tag{17}$$

which can be written as an ARCH process

$$R_t = \mu + \varepsilon_t, \quad \sigma_t^2 = \omega + \psi(L)\varepsilon_t^2, \quad \varepsilon_t \sim N(0, \sigma_t^2), \tag{18}$$

the coefficients of the lag polynomial $\psi(L)$ sum to one but decrease exponentially to zero. For the class of IGARCH processes, the shocks of the innovations ε_t on the level of the conditional variance σ_τ^2 have a strong persistence $\forall \tau > t$, which is not consistent with what is empirically observed. Thus, the occurrence of IGARCH(1,1) processes can be considered as a large sample artefact of a more complex phenomenon.

The IGARCH process is generalized with the class of long-memory ARCH, henceforth LM-ARCH, processes introduced by Robinson [45], and defined as:

$$R_t = \mu + \varepsilon_t, \quad \sigma_t^\delta = \omega + \psi(L)\varepsilon_t^\delta, \quad \varepsilon_t \sim N(0, \sigma_t^2), \tag{19}$$

where $\psi(L) = \sum_{i=1}^{\infty} \psi_i L^i$ is an infinite order lag polynomial the coefficients of which are positive and have asymptotically the following hyperbolic rate of decay $\psi_j = O\left(j^{-(1+d)}\right)$, and $\delta > 0$ is a parameter. Unlike IGARCH(1,1) processes, the persistence of the variations of the innovations on the volatility decays slowly. However, there is no stationary solutions to the equations defining a long-memory ARCH process, see e.g., [19], [28], [14], the only exception being the long-memory linear ARCH process introduced by Giraitis, Robinson and Surgailis [18]. Granger and Ding [22] and other authors considered the occurrence of long-range dependence in asset price volatilities.

2.3 Multivariate Analysis

The multivariate properties of volatility processes can be analyzed by considering the 'co-volatility' processes. The volatility processes associated with a conditional mean process R_t can be represented by its absolute value $|R_t|$ or the squared returns process R_t^2. Thus, the co-volatility of the bivariate processes $(R_{1,t}, R_{2,t})$ can be represented by the processes $\sqrt{|R_{1,t}R_{2,t}|}$ or $R_{1,t}R_{2,t}$, although only the first process is positive. Empirical evidence on asset price series, e.g., FX rates reported on Table 1, has shown that several time series share a common degree of long-range dependence in their volatilities and co-volatilities.

Table 1. Estimation of the fractional degree of integration for the series of absolute returns on Pound-Dollar $|R_{1,t}|$, Deutschmark-Dollar $|R_{2,t}|$, squared returns $R_{1,t}^2$, $R_{2,t}^2$, and the co-volatilities $\sqrt{|R_{1,t}R_{2,t}|}$ and $R_{1,t}R_{2,t}$ for the period April 1979 - January 1997. We use here the Gaussian estimator defined in (14). Asymptotic S.E. $(2\sqrt{m})^{-1}$ are between parentheses.

m	$\|R_{1,t}\|$	$\|R_{2,t}\|$	$\sqrt{\|R_{1,t}R_{2,t}\|}$
$T/4$	0.2385 (0.0147)	0.2312 (0.0147)	0.2413 (0.0147)
$T/8$	0.3071 (0.0207)	0.3219 (0.0207)	0.3230 (0.0207)
$T/16$	0.4113 (0.0293)	0.4073 (0.0293)	0.4393 (0.0293)
m	$R_{1,t}^2$	$R_{2,t}^2$	$R_{1,t}R_{2,t}$
$T/4$	0.1569 (0.0147)	0.1478 (0.0147)	0.1397 (0.0147)
$T/8$	0.2312 (0.0207)	0.2119 (0.0207)	0.2073 (0.0207)
$T/16$	0.2770 (0.0293)	0.2787 (0.0293)	0.2952 (0.0293)

A multivariate analysis of long-range dependent volatility processes can be carried by considering the parametric framework of the class of multivariate long-memory ARCH processes, introduced by Teyssière [50,51], and defined as:

$$\boldsymbol{R_t} = \boldsymbol{m(R_t)} + \boldsymbol{\varepsilon_t}, \quad \boldsymbol{\varepsilon_t} \sim \text{i.i.d.}\, \boldsymbol{N(0, \Sigma_t)}, \tag{20}$$

where $\boldsymbol{m(R_t)}$ denotes the vector regression function, $\boldsymbol{\varepsilon_t}$ is a n-dimensional vector of Gaussian error terms with conditional covariance matrix $\boldsymbol{\Sigma_t}$. The typical element $s_{ij,t}$ of $\boldsymbol{\Sigma_t}$ being either

$$s_{ij,t} = \frac{\omega_{ij}}{1-\beta_{ij}(1)} + \left(1 - \frac{(1-\phi_{ij}L)(1-L)^{d_{ij}}}{1-\beta_{ij}L}\right)\varepsilon_{i,t}\varepsilon_{j,t} \quad i,j = 1,\dots,n, \tag{21}$$

or

$$s_{ij,t} = \sum_{k=1}^{\infty} \frac{B(p_{ij}+k-1, d_{ij}+1)}{B(p_{ij}, d_{ij})} \varepsilon_{i,t-k}\varepsilon_{j,t-k}, \quad i,j = 1,\dots,n, \tag{22}$$

i.e., both conditional variances and covariances are modeled as LM-ARCH processes, which differ by different parameterizations: (21) is termed as fractionally integrated GARCH, see [1], while (22) defines the long-memory ARCH devised by Ding and Granger [10]. This class of multivariate LM-ARCH models has a few restrictions: the conditions on the parameters insuring that the matrix $\boldsymbol{\Sigma_t}$ is positive definite have to be implemented numerically in the estimation procedure. Furthermore, the number of parameters increases quickly with the dimension of the vector process, so that so far only three-dimensional models have been estimated, see [51]. However, empirical estimation results have shown that the conditional variances and covariances of several asset prices returns share the same degree of long-memory, an interesting property which stimulated further research producing the theoretical models presented later in this chapter.

2.4 Change-Point Processes

Volatility processes differ from standard long-range dependent processes: while long-range dependent time series exhibit local trends, the proxy of volatility processes, e.g., the absolute returns $|R_t|$ or the squared returns R_t^2 do not contain such a trend. Figure 1 below displays the absolute value of returns on the FTSE 100 index, which is not trended, although the estimated degree of long-memory with Robinson's [47] Gaussian estimator yields $d = 0.33$.

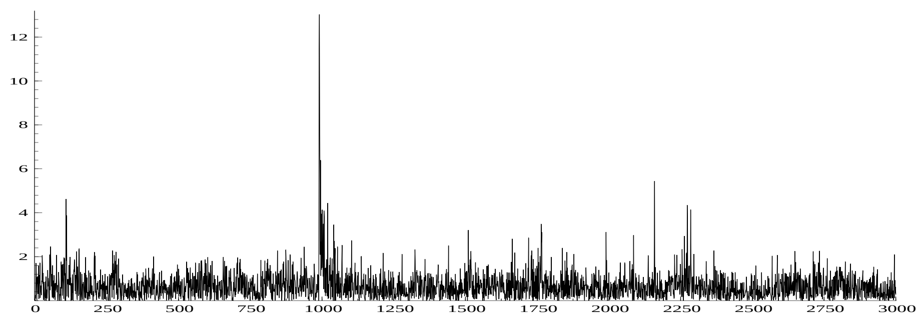

Fig. 1. Absolute returns on the FTSE 100 index.

Mikosch and Stărică [42,43] have shown that the ACF of the absolute value of a non-homogenous GARCH(1,1) process, i.e., a GARCH(1,1) process with changing coefficients, has a hyperbolic rate of decay which resembles the one of a long-range dependent process. We consider as example the following change-point GARCH(1,1) process defined as:

$$y_t = \mu + \varepsilon_t, \quad \sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2, \quad \varepsilon_t \sim N(0, \sigma_t^2), \tag{23}$$

where the parameters ω, β and α change as follows:

DGP 1: a GARCH(1,1) process with change point in the middle of the sample, such that the unconditional variance $\sigma^2 = \omega/(1-\alpha-\beta)$ remains unchanged ($\sigma^2 = 0.25$)

$$\begin{aligned} &\omega = 0.1, \quad \beta = 0.3, \quad \alpha = 0.3 \text{ for } t = 1, \ldots, [T/2] \\ &\omega = 0.15, \quad \beta = 0.25, \quad \alpha = 0.15 \text{ for } t = [T/2]+1, \ldots, T \end{aligned} \tag{24}$$

DGP 2: a GARCH(1,1) process with change in the middle of the sample, with change in the unconditional variance of the process,

$$\omega = 0.1, \quad \beta = 0.3, \quad \alpha = 0.3 \text{ for } t = 1, \dots, [T/2] \ \ (\sigma^2 = 0.25) \tag{25}$$
$$\omega = 0.15, \quad \beta = 0.65, \quad \alpha = 0.25 \text{ for } t = [T/2]+1, \dots, T \ \ (\sigma^2 = 1.5) \tag{26}$$

DGP 3: a smooth transition GARCH(1,1) process, such that the parameters $\omega(t)$, $\beta(t)$ and $\alpha(t)$ change smoothly, i.e.,

$$\begin{aligned} \omega(t) &= 0.1 + 0.05F(t, [T/2]), \quad \beta(t) = 0.3 + 0.35F(t, [T/2]), \\ \alpha(t) &= 0.3 - 0.05F(t, [T/2]), \quad \gamma = 0.05, \end{aligned} \tag{27}$$

where $F(t, k) = (1 + \exp(-\gamma(t - k)))^{-1}$, γ is a strictly positive parameter which governs the smoothness of the change. If γ becomes very large, this DGP reduces to DGP 2.

Table 2. Tests for long-range dependence on the absolute value of GARCH process with change-point in the middle of the sample. $T = 500$. Test size 5%.

	DGP 1			DGP 2			DGP 3		
q	KPSS	V/S	R/S	KPSS	V/S	R/S	KPSS	V/S	R/S
0	0.2015	0.2770	0.2995	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.1470	0.1912	0.1785	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.1203	0.1443	0.1218	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.0874	0.0918	0.0601	1.0000	0.9998	0.9996	1.0000	0.9994	0.9991
10	0.0735	0.0674	0.0356	0.9993	0.9981	0.9891	0.9994	0.9979	0.9858
20	0.0632	0.0470	0.0188	0.9945	0.9819	0.8596	0.9978	0.9817	0.8285
30	0.0567	0.0325	0.0076	0.9846	0.9274	0.4844	0.9930	0.9361	0.4112

Table 2 displays the simulation results of the various tests for long-range dependence in the absolute returns generated by the change-point GARCH processes defined above. Similar results are obtained when considering the series of squares of a non-homogeneous GARCH(1,1) processes. Thus, the tests proposed in [37], [36] and [16] can wrongly detect the presence of long-range dependence in the volatility process, while the true DGP is a non-homogeneous GARCH process with a non-constant unconditional variance. However, when the unconditional variance is constant, the power of these tests tends to their size, a statistical property which is also observed for change-point tests, see Kokoszka and Teyssière [32]. We observe that the R/S statistic is more sentitive to the truncation order q than the other statistics. Furthermore, Fig. 2 below shows absolute returns of a series generated by DGP 2. Although standard tests and estimators detect the presence of long-range dependence this series is not trended. The class of non-homogeneous GARCH(1,1) processes is also appropriate for fitting asset prices returns.

There is a substantial literature on change-point processes, interested readers are referred to [5] and [8] for complete surveys. Most of these tests are concerned

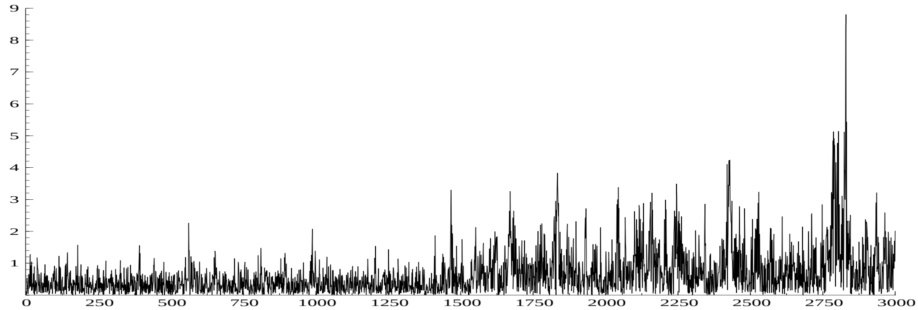

Fig. 2. Absolute value of the realization of a change-point GARCH process.

with change-point in the conditional mean processes, while we are interested here in conditional variance processes, although one can use, without theoretical foundations, these change-point tests for conditional mean processes to the volatilities and co-volatility proxy processes, i.e., $R_{1,t}^2$, $|R_{1,t}|$, $R_{1,t}R_{2,t}$, and $\sqrt{|R_{1,t}R_{2,t}|}$. We consider in this survey the tests for change point in conditional variance, proposed by Kokoszka and Leipus [34], Horváth, Kokoszka and Teyssière [25] and Kokoszka and Teyssière [32].

Kokoszka and Leipus [34] proposed a CUSUM based estimator for change-point in the class of ARCH(∞) processes at unknown time t. This estimator is defined by:

$$\hat{t} = \min\left\{t : |C_t| = \max_{1\le j\le T} |C_j|\right\}, \tag{28}$$

where

$$C_t = \frac{t(T-t)}{T^2}\left(t^{-1}\sum_{j=1}^{t} R_j^2 - \frac{1}{T-t}\sum_{j=t+1}^{T} R_j^2\right). \tag{29}$$

Horvath et al. [25] proposed several tests for change-point in ARCH sequences, based on the empirical process of squared residuals. Berkes and Horváth [4] analyzed the empirical process of squared residuals for GARCH(p,q) sequences. According to Kokoszka and Teyssière [32], some of these asymptotic tests work well when considering the squared residuals for GARCH(1,1) sequences although bootstrap tests have always the correct size and are then more reliable.

We consider here a GARCH(1,1) model fitted on the simulated returns, i.e.,

$$R_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma_t^2), \quad \sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2, \tag{30}$$

and we denote by $\hat{\varepsilon}_t^2$ the sequence of squared standardized residuals for this GARCH(1,1) model.

The first statistic is a Kolmogorov-Smirnov type statistic. For $1 \le k \le T$, define

$$\hat{T}(k,t) = \sqrt{T}\,\frac{k}{T}\left(1-\frac{k}{T}\right)\left|\hat{F}_k(t) - \hat{F}_k^*(t)\right|, \tag{31}$$

with

$$\hat{F}_k(t) = \frac{1}{k}\#\{i \le k : \hat{\varepsilon}_i^2 \le t\}, \quad \hat{F}_k^*(t) = \frac{1}{T-k}\#\{i > k : \hat{\varepsilon}_i^2 \le t\}. \tag{32}$$

The K-S statistic is defined as

$$\hat{M} = \sup_{0\le t\le\infty} \max_{1\le k\le T} |\hat{T}(k,t)|. \tag{33}$$

According to [32], correct inference is obtained by using bootstrap based inference. Horvath et al. [25] proposed also a Cramér-von Mises statistic:

$$\hat{B} = \int_0^1 \left\{ \frac{1}{T} \sum_{i=1}^{T} [\hat{T}([Ts], \hat{\varepsilon}_i^2)]^2 \right\} ds. \tag{34}$$

The distribution function of B can be derived from Blum, Kiefer and Rosenblatt [6]. Kokoszka and Teyssière [32] have shown that this asymptotic test provides correct inference.

3 Interaction Models

The class of models considered here differ from standard microeconomic models as we consider that agents are heterogeneous and do not act independently on the markets, but their beliefs and actions are affected by the predominant opinion among market participants. Keynes pointed out that individuals trades are concerned about what 'market sentiment' is rather than about fundamental values. We consider here equilibrium models, thus we rule out the case of the intra-day prices, which are not equilibrium prices but result from the content of book orders.

If the markets are efficient, the expected price $E(P_{t+1})$ of an asset at time $t+1$ conditional on the information set I_t is given by:

$$E(P_{t+1}|I_t) = P_t. \tag{35}$$

In our model, agents do not consider markets to be efficient and assume that they can predict the next price P_{t+1}. Chartists assume that the exchange rate P_{t+1} is a convex linear function of the previous prices, i.e.,

$$E^c\,(P_{t+1}|I_t) = \sum_{j=0}^{M^c} h_j P_{t-j}, \quad \text{with} \quad \sum_{j=0}^{M^c} h_j = 1, \tag{36}$$

where h_j, $j = 0, \ldots, M^c$ are constants, M^c is the memory of the chartists, while fundamentalists forecast the next price as:

$$E^f\,(P_{t+1}|I_t) = \bar{P}_t + \sum_{j=1}^{M^f} \nu_j (P_{t-j+1} - \bar{P}_{t-j}), \tag{37}$$

where ν_j, $j = 1, \ldots, M^f$ are positive constants, representing the degree of reversion to the fundamentals, M^f is the memory of the fundamentalists. This series of 'fundamentals' $\bar{P}_t$, which can be thought as the price if it were only to be explained by a set of relevant variables, is assumed to follow a random walk:

$$\bar{P}_t = \bar{P}_{t-1} + \varepsilon_t, \quad \text{with} \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2). \tag{38}$$

Individuals i, $i = 1, \ldots, N$ have a standard mean-variance utility function:

$$U(W_{t+1}^i) = E(W_{t+1}^i) - \lambda V(W_{t+1}^i), \tag{39}$$

where λ denotes the risk aversion coefficient, $E(.)$ and $V(.)$ denote the expectation and variance operators. Agents have the possibility of investing at home in a risk free asset or investing abroad in a risky asset.

Denote by ρ_t the foreign interest rate, by d_t^i the demand by the i^{th} individual for foreign currency, and by r the domestic interest rate. The exchange rate P_t and the foreign interest rate ρ_t are considered by agents as independent random variables, with

$$\rho_t \sim N(\rho, \sigma_\rho^2) \quad \text{with} \quad \rho_t > r. \tag{40}$$

Hence, the cumulated wealth of individual i at time $t+1$, W_{t+1}^i is given by:

$$W_{t+1}^i = (1 + \rho_{t+1}) P_{t+1} d_t^i + (W_t^i - P_t d_t^i)(1 + r). \tag{41}$$

Thus, we have:

$$E(W_{t+1}^i | I_t) = (1 + \rho) E^i(P_{t+1} | I_t) d_t^i + (W_t^i - P_t d_t^i)(1 + r), \tag{42}$$

$$V(W_{t+1}^i | I_t) = (d_t^i)^2 \zeta_t \quad \text{where} \quad \zeta_t = V\left(P_{t+1}(1 + \rho_{t+1})\right). \tag{43}$$

Demand d_t^i is found by maximizing utility. First order condition gives

$$(1 + \rho) E^i(P_{t+1} | I_t) - (1 + r) P_t - 2\zeta_t \lambda d_t^i = 0, \tag{44}$$

where $E^i(.|I_t)$ denotes the expectation of an agent of type i. Let k_t be the proportion of fundamentalists at time t, the market demand is:

$$d_t = \frac{(1 + \rho)\left(k_t E^f(P_{t+1} | I_t) + (1 - k_t) E^c(P_{t+1} | I_t)\right) - (1 + r) P_t}{2\zeta_t \lambda}. \tag{45}$$

Now consider the exogenous supply of foreign exchange X_t, then the market is in equilibrium if aggregate supply is equal to aggregate demand, i.e., $X_t = d_t$, which gives

$$P_t = \frac{1 + \rho}{1 + r}\left(k_t E^f(P_{t+1} | I_t) + (1 - k_t) E^c(P_{t+1} | I_t)\right) - \frac{2\zeta_t \lambda X_t}{1 + r}. \tag{46}$$

We assume that $2\zeta_t \lambda X_t / (1 + \rho) = \gamma \bar{P}_t$. If $M^f = M^c = 1$, then the equilibrium price is given by

$$P_t = \frac{k_t - \gamma}{A} \bar{P}_t - \frac{k_t \nu_1}{A} \bar{P}_{t-1} + \frac{(1 - k_t) h_1}{A} P_{t-1}, \tag{47}$$

with

$$A = \frac{1+r}{1+\rho} - (1-k_t)h_0 - k_t\nu_1. \tag{48}$$

Thus, when k_t jumps from zero to one, our so called 'Havana-India' model resembles a change-point process in the conditional mean. Since the process k_t is likely to take all values between 0 and 1, it is of interest to study the effects of the evolution of the process k_t on the occurrence of long-range dependence in the volatility of the series generated by the microeconomic model.

We consider a multivariate extension of this model, i.e., the joint modeling of a bivariate process $(P_{1,t}, P_{2,t})$. Both exchange rates depend on a pair of foreign interest rates $(\rho_{1,t}, \rho_{2,t})$. Our bivariate model then becomes:

$$\begin{pmatrix} P_{1,t} \\ P_{2,t} \end{pmatrix} = \begin{pmatrix} \frac{k_t-\gamma}{A_1}\bar{P}_{1,t} - \frac{k_t\nu_{1,1}}{A_1}\bar{P}_{1,t-1} + \frac{(1-k_t)h_{1,1}}{A_1}P_{1,t-1} \\ \frac{k_t-\gamma}{A_2}\bar{P}_{2,t} - \frac{k_t\nu_{2,1}}{A_2}\bar{P}_{2,t-1} + \frac{(1-k_t)h_{2,1}}{A_2}P_{2,t-1} \end{pmatrix}, \tag{49}$$

with

$$A_i = \frac{1+r}{1+\rho_i} - (1-k_t)h_{i,0} - k_t\nu_{i,1}. \tag{50}$$

We assume that the bivariate process of fundamentals $(\bar{P}_{1,t}, \bar{P}_{2,t})$ displays some form of positive correlation, i.e.,

$$\begin{gathered} \begin{pmatrix} \bar{P}_{1,t} \\ \bar{P}_{2,t} \end{pmatrix} = \begin{pmatrix} \bar{P}_{1,t-1} \\ \bar{P}_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}, \\ \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{1,1}^2 & \sigma_{1,2} \\ \sigma_{1,2} & \sigma_{2,2}^2 \end{pmatrix}\right], \quad \sigma_{1,2} > 0. \end{gathered} \tag{51}$$

In the simulation study, we set $\sigma_{1,2}$ so that the coefficient of correlation between the two processes $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ is equal to 0.75. This choice has been motivated by the estimation results in [50] for the bivariate long-memory ARCH processes, where the coefficient of correlation in the conditional covariance matrix $\boldsymbol{\Sigma_t}$ has been found equal to 0.75. As we will see in Sect. 4, the assumption of a positive correlation is crucial if we are interested in the co-volatility processes $\sqrt{|R_{1,t}R_{2,t}|}$ and $R_{1,t}R_{2,t}$: in that case these co-volatility processes have exactly the same degree of long-memory as the processes $|R_{1,t}|$, $|R_{2,t}|$ and $R_{1,t}^2$, $R_{2,t}^2$ respectively, in accordance with the empirical findings of [50]. In [29], we assume $\sigma_{1,2} = 0$, and simulation results are less satisfactory than the current ones, as the degree of LRD for the series $\sqrt{|R_{1,t}R_{2,t}|}$ is slightly higher than the ones for the series $|R_{1,t}|$, $|R_{2,t}|$.

We also assume that the process k_t is the same for both markets, i.e., the proportion of fundamentalists is the same for both currencies. This assumption is consistent with the one that fundamentals for both series are correlated, i.e., both FX markets are linked. This is a reasonable assumption if we consider that both currencies belong to the same 'target-zone', see [12].

We consider here several types of processes for $\{k_t\}_{t=1}^T$. The first one is the epidemiologic process introduced by Hans Föllmer and used in [31,29,30], where agents interact and communicate their beliefs on the next period forecast through Föllmer's epidemiologic process.

Let N be the total number of agents and ϑ_t be the number of agents with a fundamentalist forecast at time t. We assume that pairs of agents meet at random and that the probability that the first agent is converted to the opinion of the second one is equal to $(1-\delta)$. Furthermore, each agent can independently change his opinion with probability ξ, so that the process is not trapped in the extremes, i.e., agents are either all chartists or all fundamentalists.

Given that the state of the process is summarized by the value of ϑ_t, its evolution is defined by the following transition matrix:

$$\Pr(\vartheta, \vartheta+1) = \left(1 - \frac{\vartheta}{N}\right)\left(\xi + (1-\delta)\frac{\vartheta}{N-1}\right), \tag{52}$$

$$\Pr(\vartheta, \vartheta-1) = \frac{\vartheta}{N}\left(\xi + (1-\delta)\frac{N-\vartheta}{N-1}\right), \tag{53}$$

$$\Pr(\vartheta, \vartheta) = 1 - \Pr(\vartheta, \vartheta+1) - \Pr(\vartheta, \vartheta-1). \tag{54}$$

For this epidemiologic process, the proportion of fundamentalists and the forecasts of agents does not depend on the past performance of forecasts functions. For that reason, we can consider a diffusion process for k_t which depends on the accuracy of the forecast functions in the recent periods: the probability of choosing a particular forecast function depends on its comparative performance over the competing forecast function. We can use Theil's [52] U statistic as measure of forecast accuracy over the last M periods:

$$U_M^j = \sqrt{\frac{M^{-1}\sum_{l=1}^{M} w_l \left(P_{t-l} - E^j(P_{t-l}|I_{t-1-l})\right)^2}{M^{-1}\sum_{l=1}^{M} w_l P_{t-l}^2}}, \quad j \in \{c, f\}, \quad \sum_l w_l = 1, \tag{55}$$

M being the learning memory of agents, the weights $w_l, l = 1, \ldots, M$ representing the relative importance of the forecast errors at time $t-l$. We choose here an exponential choice function $g^j(\cdot)$ for the forecast function $E^j(\cdot)$ defined by:

$$g^j(t) = \exp(-\varUpsilon U_M^j), \quad \varUpsilon > 0, \quad j \in \{c, f\}, \tag{56}$$

the parameter $\varUpsilon$ is called the "intensity of choice". At time t, agents will chose with probability $\pi^f(t)$ the fundamentalist forecast function, where

$$\pi^f(t) = \frac{g^f(t)}{g^f(t) + g^c(t)}, \tag{57}$$

the probability of choosing the chartist forecast function is $\pi^c(t) = 1 - \pi^f(t)$. For the bivariate process, the probability of choosing the fundamentalist forecast function is given by averaging the two choice functions for both markets.

Let ϑ_t/N be the proportion of fundamentalists resulting from either the epidemiologic process or the learning process. We assume that agents observe this proportion with error, i.e., agent i observe $k_{i,t}$ defined as:

$$k_{i,t} = \frac{\vartheta_t}{N} + \varepsilon_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \sim N(0, \sigma_\vartheta^2). \tag{58}$$

If agent i observe $k_{i,t} \geq 0.5$, then he will make a fundamentalist forecast, otherwise he will make a chartist forecast. The proportion k_t of agents making a fundamentalist forecast is then given by:

$$k_t = N^{-1} \# \left\{ i : k_{i,t} \geq \frac{1}{2} \right\}. \tag{59}$$

For the epidemiologic case, the herding behavior of the process k_t into the extremes depends on ξ and σ_ϑ, while it depends on Υ and σ_ϑ for the process based on the forecasts accuracy. For both processes, the parameter σ_ϑ measures the accuracy of observation of the proportion of fundamentalists; see (58). If σ_ϑ becomes smaller, the prevailing opinion is observed with more accuracy, which results in massive swings of opinion.

As we will see in the next section, these parameters govern the level of long-range dependence in the volatility of the simulated returns.

4 Simulation Study

We simulated 10.000 replications of our microeconomic models. We considered samples of 1500 observations. The models generates the empirical properties of asset prices returns. The series of asset returns R_t do not display dependence, the average estimated value for d is $\hat{d} = 0.002$ for the series R_t. When the sample size increases from 750 to 1500, the estimated value for d increases from $\hat{d} = 0.20$ to $\hat{d} = 0.28$. When estimating the parameters of a GARCH processes on the series of 750 observations, we get $\hat{\alpha} = 0.04$ and $\hat{\beta} = 0.74$, while for the series of 1500 observations, $\hat{\alpha} = 0.055$ and $\hat{\beta} = 0.88$: the model replicates the empirical property of occurrence of IGARCH processes when the sample size increases, see [30]. The occurrence of long-range dependence in asset prices volatility might be the consequence of several changes in regime in the price process.

The level of long-range dependence d of the simulated processes increases when we reduce the value of σ_ϑ, i.e., when the proportion of fundamentalists is observed with more accuracy: in that case the process k_t herds into the extremes. The level of long-range dependence is linked to the swings in the predominant opinion which make the price process defined by (47) switching between two regimes.

The assumption of a positive correlation between the fundamentals proved to be important. In [29], we assume that there is no correlation between the two processes $(\varepsilon_{1,t}, \varepsilon_{2,t})$, i.e., $\sigma_{1,2} = 0$. As a consequence, the estimated level of long-range dependence in the co-volatility process $\sqrt{|R_{1,t}R_{2,t}|}$ was slightly higher than the one of the volatility processes $|R_{1,t}|$ and $|R_{2,t}|$. Furthermore, for this uncorrelated setting, the co-volatility process $R_{1,t}R_{2,t}$ does not display

Table 3. Tests for long-range dependence on the absolute value of Simulated returns, R_t, absolute returns $|R_t|$ and squared returns R_t^2. $T = 1500$. Test size 5%.

	$R_t,\ P(d=0)$			$\lvert R_t\rvert,\ P(d>0)$			$R_t^2,\ P(d>0)$		
q	KPSS	V/S	R/S	KPSS	V/S	R/S	KPSS	V/S	R/S
0	0.9369	0.9358	0.9343	0.9460	0.9772	0.9720	0.9440	0.9733	0.9739
1	0.9389	0.9376	0.9369	0.9408	0.9739	0.9687	0.9349	0.9674	0.9655
2	0.9395	0.9408	0.9388	0.9375	0.9687	0.9642	0.9323	0.9648	0.9609
3	0.9402	0.9486	0.9414	0.9343	0.9635	0.9622	0.9271	0.9609	0.9557
4	0.9395	0.9512	0.9421	0.9297	0.9616	0.9609	0.9226	0.9577	0.9531
5	0.9388	0.9577	0.9453	0.9271	0.9590	0.9590	0.9219	0.9551	0.9512
10	0.9375	0.9629	0.9512	0.9161	0.9486	0.9473	0.9076	0.9408	0.9421
15	0.9395	0.9603	0.9531	0.8946	0.9375	0.9388	0.8875	0.9284	0.9336
20	0.9369	0.9649	0.9557	0.8777	0.9265	0.9271	0.8719	0.9167	0.9232
25	0.9395	0.9649	0.9557	0.8595	0.9031	0.9161	0.8491	0.8907	0.9083
30	0.9375	0.9681	0.9551	0.8270	0.8823	0.9024	0.8296	0.8615	0.8927

Table 4. Gaussian estimates of d for the bivariate series of simulated absolute returns $|R_{1,t}|$, $|R_{2,t}|$, $\sqrt{|R_{1,t}R_{2,t}|}$. (Monte Carlo S.E. in parenthesis.) $T = 1500$. m_{opt} denotes the optimal bandwidth.

m	$\lvert R_{1,t}\rvert$	$\lvert R_{2,t}\rvert$	$\sqrt{\lvert R_{1,t}R_{2,t}\rvert}$
m_{opt}	0.2771 (0.1127)	0.2766 (0.1124)	0.2793 (0.1130)
84	0.2984 (0.0965)	0.2978 (0.0975)	0.3008 (0.0978)
108	0.2660 (0.0880)	0.2653 (0.0873)	0.2670 (0.0883)
132	0.2421 (0.0811)	0.2411 (0.0814)	0.2432 (0.0821)
156	0.2244 (0.0753)	0.2232 (0.0757)	0.2250 (0.0755)

Table 5. Gaussian estimates of d for the bivariate series of simulated squared returns $R_{1,t}^2$, $R_{2,t}^2$, $R_{1,t}R_{2,t}$. (Monte Carlo S.E. in parenthesis.) $T = 1500$. m_{opt} denotes the optimal bandwidth.

m	$R_{1,t}^2$	$R_{2,t}^2$	$R_{1,t}R_{2,t}$
m_{opt}	0.2582 (0.1076)	0.2592 (0.1096)	0.2121 (0.1009)
84	0.2851 (0.0939)	0.2852 (0.0956)	0.2462 (0.0928)
108	0.2534 (0.0855)	0.2541 (0.0857)	0.2176 (0.0835)
132	0.2304 (0.0788)	0.2308 (0.0793)	0.1978 (0.0775)
156	0.2137 (0.0723)	0.2130 (0.0736)	0.1822 (0.0706)

any long-range dependence. With the assumption that $\sigma_{1,2} > 0$, the simulated co-volatility process $R_{1,t}R_{2,t}$ displays long-memory, the degree of which is close to the one of the series $R_{1,t}^2$ and $R_{2,t}^2$, as empirically observed, see Tables 4 and 5.

From Table 6, we can see that the V/S and R/S 'pox-plot' estimation results do not differ too much from the ones provided by the Gaussian estimator [47].

We report here simulation results for the CVM and K-S change-point tests. Interested readers are referred to [31,30] for the performance of the test by Kokoszka and Leipus [34]. Given that the asymptotic K-S test does not have the correct size, we resort to bootstrap based inference for this test, the number

Table 6. "Pox-plot" estimates of d based on the squared returns series R_t^2. (Monte Carlo S.E. in parenthesis.) $T = 1500$.

	R/S estimate of d	V/S estimate of d	KPSS estimate of d
d	0.2506 (0.0791)	0.2604 (0.0993)	0.3324 (0.1182)

of bootstraps B is set to 399 for all replications. For a test of size 5%, the CVM test rejects 22.97% of the times the null hypothesis of no change-point, while the K-S test rejects this null hypothesis 20.58% of the times. When interpreting these results, we have to keep in mind that these tests have been devised for processes with a single change-point in the conditional variance, and that we apply them to the first-difference of non-standard conditional mean processes, which can have multiple changes in regime.

Figure 3 displays the absolute value of a series of simulated returns generated by the model. This series resembles the series of absolute returns on asset prices, i.e., it does not have a trend, although the ACF of this series displayed LRD-type behavior, see Fig. 4. In Kokoszka and Teyssière [32] and Kirman and Teyssière [31], we used the wavelet estimator by Veitch and Abry [53] for estimating the degree of LRD of several asset prices volatilities and the volatility process generated by our model. Wavelet analysis is of interest as this multi-

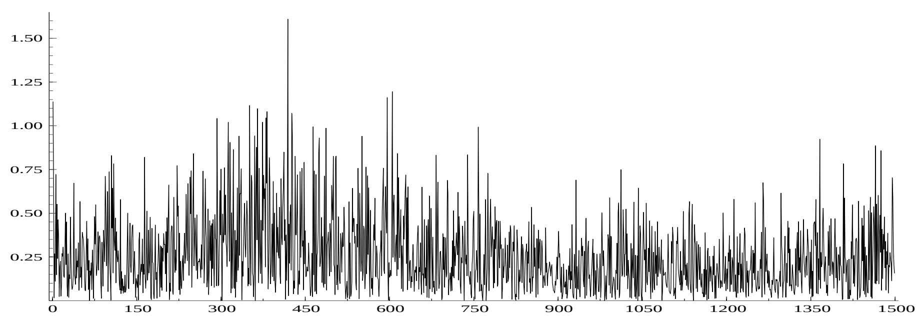

Fig. 3. Absolute value of a series of returns produced by the microeconomic model.

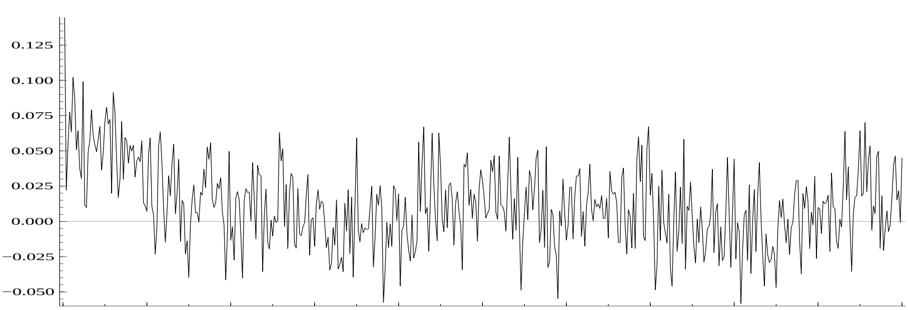

Fig. 4. ACF of the absolute value of a series of returns produced by the model.

resolution analysis is unaffected by changes in the location parameter of a time series and is then able to distinguish between genuine long-range dependence and spurious long-range dependence caused by changes in regimes.

For both real data and series simulated by our model, we observed that the estimated degree of LRD with the wavelet estimator is far lower than the one obtained with the Whittle estimator. The degree of LRD for the absolute returns $|R_{1,t}|$ on British Pound-US dollar drops from 0.41 when estimated with the local Whittle estimator to 0.12 when estimated with the wavelet estimator, while the degree of LRD for the absolute returns $|R_{2,t}|$ on German Deutschmark-US dollar falls from 0.40 to 0.12 respectively. We observe the same changes for the degrees of LRD for the other empirical and simulated volatility and co-volatility processes, i.e., $R_{1,t}^2$, $R_{2,t}^2$, $\sqrt{|R_{1,t}R_{2,t}|}$ and $R_{1,t}R_{2,t}$. Furthermore, for several series, the confidence intervals for the wavelet estimates often contain the value zero. Our microeconomic models are then able to generate most of the empirical dependence properties of daily asset returns.

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