Microeconomic Models for Long–Memory in the Volatility of Financial Time Series[1]

Alan KIRMAN[2]
EHESS, Université d'Aix-Marseille III
Institut Universitaire de France & GREQAM

Gilles TEYSSIÈRE[3]
GREQAM
CORE

First Draft: February 1998
Final Version.

[1] We are greatly indebted to Hans Föllmer with whom the stochastic process used in this paper has been derived, to Raj Bhansali, Russell Davidson, Liudas Giraitis, Clive Granger, Marc Henry, Cars Hommes, Piotr Kokoszka, Remigijus Leipus, Donatas Surgailis, Bas Werker, for useful comments and discussions. We thank the editor, Bruce Mizrach, for comments which improve the presentation, and seminar participants to the Weierstrass Institute of Applied Analysis and Stochastics in Berlin, October 1998; the Department of Probability and Mathematical Statistics of Charles University in Prague, November 1998; the Division of Statistics in the Department of Mathematics of the University of Liverpool, February 1999; the 1999 Econometric Society European Meeting in Santiago de Compostela, August 1999; the Society for Nonlinear Dynamics and Econometrics Annual Symposium in Atlanta, March 2000; the CeNDEF seminar at the Free University of Amsterdam, February 2001; the 8^{th} EWGFM Meeting in Vilnius, May 2001, for comments. We also thank Olsen & Associates and Rakhal Dave for providing some of the data used in this paper. Both authors will blame each other for the remaining mistakes.

[2] **Correspondence:** GREQAM, Centre de la Vieille Charité, 2, rue de la Charité, F-13002 Marseille, FRANCE. **E-mail:** kirman@ehess.cnrs-mrs.fr

[3] **Correspondence:** GREQAM, Centre de la Vieille Charité, 2, rue de la Charité, F-13002 Marseille, FRANCE. **E-Mail:** gilles@ehess.cnrs-mrs.fr
Home Page: http://members.aol.com/gteyssiere

Abstract

We show that a class of microeconomic behavioral models with interacting agents, derived from Kirman (1991, 1993), can replicate the empirical long–memory properties of the two first conditional moments of financial time series. The essence of these models is that the forecasts and thus the desired trades of the individuals in the markets are influenced, directly, or indirectly by those of the other participants. These "field effects" generate "herding" behaviour which affects the structure of the asset price dynamics. The series of returns generated by these models display the same empirical properties as financial returns: returns are $I(0)$, the series of absolute and squared returns display strong dependence, while the series of absolute returns do not display a trend. Furthermore, this class of models is able to replicate the common long–memory properties in the volatility and co-volatility of financial time series, revealed by Teyssière (1997, 1998a). These properties are investigated by using various model independent tests and estimators, i.e., semiparametric and nonparametric, introduced by Lo (1991), Kwiatkowski, Phillips, Schmidt and Shin (1992), Robinson (1995), Lobato and Robinson (1998), Giraitis, Kokoszka Leipus and Teyssière (2000, 2001). The relative performance of these tests and estimators for long–memory in a non-standard data generating process is then assessed.
Keywords: long–memory, microeconomic models, field effects, semiparametric tests, conditional heteroskedasticity,
JEL Classification: C15, C22, D40.

1 Introduction

Over time, a clearer picture of some of the statistical features of time series of asset prices has emerged. An extensive literature has been developed on the subject of testing for these features. However, few models based on microeconomic behavior which actually generate these characteristics have been proposed. This paper presents a class of models in which agents interact stochastically on a financial market and which do, in fact, generate the statistical characteristics found empirically.

The dynamics of the two first conditional moments of asset prices are rather complex. The mean series are, in general, non-stationary, have unit roots at daily frequency, and exhibit bubbles, with a clustering of large deviations, while the volatility series display a form of significant dependence between very distant observations called long–range dependence or long–memory.[1]

The econometric has literature focused on the statistical tests and models for testing and representing these empirical features. There is a substantial literature on unit roots, non-stationarity tests, and their refinements: see Dickey and Fuller (1979), Sowell (1990), Robinson (1991, 1994a, 1994b), Kwiatkowski *et al.* (1992), MacKinnon (1994, 1996), Lee and Schmidt (1996), among others. Basic references for testing for bubbles are Blanchard and Watson (1982), Hamilton (1989), and Evans (1991).

As a consequence of the martingale property of daily prices P_t, the log of returns $R_t = \ln(P_t/P_{t-1})$ are uncorrelated. However their power transformation $|R_t|^\delta$, where δ is a positive real number, are characterized by a long-range serial dependence, or long–memory. This long–range dependence in the volatility can be modeled by resorting to the class of long–memory ARCH processes, introduced by Robinson (1991),[2] the long–memory stochastic volatility models,[3] the multi-factors models of Gallant, Hsu and Tauchen (1998), and the multifractal model of asset returns developed by Mandelbrot, Fisher and Calvet (1997).

The fact that the class of fractional models can capture the rich dynamics of asset prices, and their transformations, is not particularly surprising as fractals mathematics have been used for describing a large variety of complex phenomena in geophysical sciences, economics, engineering, *etc.* However, a simple descriptive approach is incomplete, as many empirical results show some regularity in the long–memory structure across economic series: Teyssière (1997, 1998a) revealed that some daily and intra-day Foreign Exchange (FX) rates returns display the same degree of long–memory in their conditional variances and covariances. Since these regularities are not fortuitous, and are presumably caused by some economic phenomenon, the obvious next step is to devise a structural model generating these statistical properties.

Structural models for long–memory are rather rare. Willinger, Paxson and Taqqu (1998) have proposed a framework explaining the self-similarity in the mean series of Ethernet traffic. Box-Steffenmaier and Smith (1996) and Byers, Davidson and Peel (1997) have considered aggregate popularity models explaining persistence in opinion polls. The results of this series of contributions are derived by making some distributional assumption on the parameters of the models, and by resorting to statistical distributions which when aggregated lead to long–memory processes. The long–memory properties of the Mandelbrot, Fisher and Calvet (1997) multifractal model rely on the multifractal distribution of the trading time.

Our approach is based on behavioural models which do not resort to the above mentioned

[1] See Mandelbrot (1963), Taylor (1986), Granger and Ding (1995, 1996).
[2] See Ding and Granger (1996), Granger and Ding (1995), Giraitis, Robinson and Surgailis (2000).
[3] See Harvey (1998), Breidt, Crato and de Lima (1998), Robinson (2001).

statistical distributions. The essence of these models is that the forecasts and thus the desired trades of the individuals in the markets are influenced, directly, or indirectly by those of the other participants. This interdependence, called a "field effect",[4] generates "herding" behaviour which affects the structure of the asset price dynamics. This research was motivated by the fact that one of the authors, (the econometrician), has been working on the long–memory properties of financial data, and was embarrassed by the lack of theoretical models explaining his empirical findings. He looked at one of the models devised by the other author (the microeconomist) he programmed in 1991. After a few changes, a convincing answer to his questions emerged.

These models replicate other empirical properties of financial data. For large samples, the simulated series of returns can be fitted by IGARCH models, see Kirman and Teyssière (2001, 2002). Furthermore, like absolute returns on asset prices, our simulated series of absolute returns do not display a trend and then differ from standard long–memory processes.

These models were originally intended to explain bubbles in asset price series and to develop an economic model to reinforce Evans' (1991) explanation of the possibility of "rational bubbles". A number of models generating herding behaviour have been proposed by Day and Huang (1990), Banerjee (1992), Bikhchandani *et al.* (1992), and Welch (1992), for example. The models proposed here rely neither on persistent erroneous beliefs nor do they emphasize convergence to a single self-fulfilling belief as in the "informational cascades" or "sunspots" literature, see e.g., Woodford (1990). Our models exhibit continual switching from one dominant belief to another and there is no convergence to a particular state. The appropriate equilibrium notion is that of a limiting distribution.

The paper is organized as follows: in section **2** we give some empirical evidence which lead us to consider structural models for long–memory. Given that the behavioral models considered here do not belong to a standard family of stochastic processes, we consider only model independent tests and estimators for long–range dependence, which are presented in section **3**. Section **4** presents these behavioral models. Simulation results are reported in section **5**, where the relative performance of these semiparametric and nonparametric tests and estimators with respect to a non-standard data generating process is discussed. Section **6** concludes.

2 The empirical properties of financial data

2.1 Long–memory processes

A stationary process Y_t is called a stationary process with long–memory if its autocorrelation function, henceforth ACF, $\rho(k)$ has asymptotically the following hyperbolic rate of decay:[5]

$$\rho(k) \sim L(k)k^{2d-1} \quad \text{as } k \to \infty \tag{2.1}$$

where $L(k)$ is a slowly varying function,[6] and $d \in (0, 1/2)$ is the parameter which governs the slow rate of decay of the ACF and then parsimoniously summarizes the degree of long-range dependence of the series. In contrast, the ACF of a "short-memory" process, such as an ARMA process, converges at an exponential rate, i.e., very quickly, to zero.

[4]See Aoki (1996) and Durlauf (1997) for surveys on models with interacting agents.

[5]See Beran (1994), Granger (1980), Granger and Joyeux (1980), Hosking (1981), and Robinson (1994a).

[6]A function $L(k)$, $k \geq 0$, is called slowly varying function if $L(\lambda k)/L(k) \to 1$ as $k \to \infty$, $\forall \lambda > 0$.

Equivalently, the spectrum $f(\lambda)$ of a stationary process with long–memory parameter d can be approximated in the neighborhood of the zero frequency as:

$$f(\lambda) \sim C\lambda^{-2d} \quad \text{as } \lambda \to 0^+ \tag{2.2}$$

where C is a finite strictly positive constant. The autocorrelations of a long–memory process are then not summable, i.e., $\sum_{k=1}^{\infty} \rho(k) = \infty$, and the spectrum of a long–memory process has a pole at frequency zero as $\lim_{\lambda \to 0^+} f(\lambda) = \infty$.

We detect long–memory and estimate the long–memory parameter d in various ways. In a first approach, we consider a class of parametric long–memory models, the simplest one being the fractional noise or $I(d)$ process, defined by:

$$(1-L)^d y_t = \varepsilon_t, \quad \varepsilon_t \sim i.i.d(0,\sigma^2), \quad (1-L)^d = \sum_{k=0}^{\infty} \psi_k L^k, \quad \psi_0 = 1,\, \psi_k = \prod_{j=1}^{k} \left(1 - \frac{1+d}{j}\right) \tag{2.3}$$

where the real parameter d is called the fractional degree of integration of the process, and $(1-L)^d$ is the fractional difference operator. This process is stationary if $d < 1/2$, mean reverting if $d < 1$, and invertible if $d > -1$.[7]

Daily prices P_t are normally found to be $I(1)$, which is a consequence of the efficient market hypothesis, i.e., $E(P_{t+1}|I_t) = P_t$, where I_t denotes the information set available at time t. While the log of returns $R_t = \ln(P_t/P_{t-1})$ are then $I(0)$, their power transformation $|R_t|^\delta$ display long–memory. Figure 1 on p 22 displays the ACF of the absolute returns on Pound-US Dollar for the period 1979-1997. The slow decay of this ACF is typical of a long–memory process, and suggests the presence of long–memory in the volatility of asset returns.

Long-range dependence in the conditional second moments was first considered by Robinson (1991), who introduced the class of long–memory ARCH processes for testing for no ARCH effects. The general form for an ARCH(∞) is:

$$R_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim D(0,1) \quad \text{with} \quad \sigma_t^2 = \sigma^2 + \sum_{j=1}^{\infty} \psi_j (g(\varepsilon_{t-j}) - \sigma^2) \tag{2.4}$$

for some $\sigma^2 > 0$, where $D(0,1)$ is a distribution with zero mean and variance equal to one, and $\sigma^2 - g(\varepsilon_t)$ is a martingale difference. If $\psi_j = 0$ for $j > p$, this model is an ARCH(p) model, while if the infinite sequence $\{\psi_j\}_{j=1}^{\infty}$ has the asymptotic hyperbolic rate of decay $\psi_j = O(j^{-(1+d)})$, then equation (2.4) defines an ARCH(∞) process with long–memory parameter d: for this process, shocks to the error terms have a persistent effect on the conditional variance.

The class of long–memory ARCH models is able to capture the hyperbolic rate of decay of the volatility autocorrelations, provided that a suitable model is selected. Figure 2 displays the empirical ACF of the absolute returns $|R_t|$ on the S&P 500 Composite Index P_t, with the averaged ACF of 2000 simulated FINGARCH-t processes estimated on these data.[8] The conditional heteroskedastic function of a FINGARCH process is defined as:

$$\sigma_t^\delta = \frac{\omega}{1-\beta(1)} + \left(1 - \frac{(1-\phi(L))\,(1-L)^d}{1-\beta(L)}\right) |\varepsilon_t + \gamma\sigma_t|^\delta \tag{2.5}$$

[7] See Odaki (1993).

[8] This picture is borrowed from Teyssière (1998b), where the FINGARCH process has been introduced.

Although this model is able to capture the hyperbolic decay of the ACF more adequately than most other long–memory ARCH models,[9] it shares the same drawbacks of the long–memory volatility models: the asymptotic normality and root-n consistency of the approximate maximum likelihood estimator are only conjectured on the basis of Monte Carlo simulations, and there might be no strictly stationary solution to the equations defining a Fractionally Integrated GARCH process, see Giraitis, Kokoszka and Leipus (2000), Kazakevičius and Leipus (2002), Kazakevičius, Leipus and Viano (2000), and Giraitis and Surgailis (2001), who contradict the assertion of Baillie *et al.* (1996). Furthermore, the unconditional variance of a long–memory FIGARCH type process does not exist, which is not consistent with what is empirically observed for several series of asset returns, see e.g., Dacorogna *et al.* (1995).

2.2 Empirical evidence

We observe some regularity in the estimated long–memory components of the volatility. As the volatility of a series R_t can be represented by its absolute value $|R_t|$, see Granger and Ding (1995), we define the "co-volatility" between the series $R_{1,t}$ and $R_{2,t}$ by the expression $\sqrt{|R_{1,t}R_{2,t}|}$. Teyssière (1997) analyzed the volatility and co-volatility of daily returns on Pound-US Dollar and Deutschmark-US Dollar by introducing the bivariate unrestricted long–memory ARCH model defined as:

$$\begin{pmatrix} R_{1,t} \\ R_{2,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} s_{11,t} & s_{12,t} \\ s_{12,t} & s_{22,t} \end{pmatrix}\right] \tag{2.6}$$

the specification of the conditional covariance matrix being either

$$s_{ij,t} = \frac{\omega_{ij}}{1-\beta_{ij}(1)} + \left(1 - \frac{(1-\phi_{ij}L)(1-L)^{d_{ij}}}{1-\beta_{ij}L}\right)\varepsilon_{i,t}\varepsilon_{j,t} \quad i,j = 1,2 \tag{2.7}$$

or

$$s_{ij,t} = \sum_{k=1}^{\infty} \frac{B(p_{ij}+k-1, d_{ij}+1)}{B(p_{ij}, d_{ij})}\varepsilon_{i,t-k}\varepsilon_{j,t-k}, \quad i,j = 1,2 \tag{2.8}$$

i.e., both conditional variances and covariances are modeled as long–memory ARCH processes. Estimation results by quasi maximum likelihood methods led us to accept the hypothesis that the three long–memory parameters are the same, i.e., $d_1 = d_2 = d_3 = 0.4187$ for the period 1979-1997. This empirical result is confirmed if we estimate the degree of long-range dependence of the volatilities and co-volatilities with Robinson's (1995) semiparametric estimator presented in the next section. We also consider in this paper the series of French Franc-US Dollar daily returns and estimate the degree of long memory of the three volatilities and three co-volatilities with the same estimator, see table 1 in the appendix. As the estimated value of $\hat{d}$ is the same for the different bandwidths m, we can then conclude that the volatilities and co-volatilities of these three FX rates share the same degree of long–memory.

We also evaluate the degree of long–memory of the volatilities and co-volatilities of the Pound-French Franc and Deutschmark-French Franc with the same estimator. Results displayed in table 2 in the appendix show that the level of long–memory depends on the FX market, however the property of a common long–memory component is preserved.

[9] Another case of perfect fit is given in Ding and Granger (1996). In other cases, the quality of the fit is not so perfect.

These regularities in the long–memory properties are certainly not fortuitous, since such a common degree of long-range dependence has also been observed for more recent series of intra-day FX rates, Pound-US Dollar, Deutschmark-US Dollar, and Yen-US Dollar. Table 3 in appendix reports the estimates of d for the 30 minutes spaced Olsen & Associates HFDF-96 series of absolute returns on the following FX rates: Dollar-Deutschmark, Dollar-Yen, Dollar-Swiss Franc, and Dollar-Pound for the year 1996.[10] Similar results on the commonality of the long–memory component have been observed with different statistical methods by Henry and Payne (1997) for high frequency FX rates, and Ray and Tsay (2000) in the US stock market. Teyssière (1998a) estimated a trivariate ARFIMA-FIGARCH on 30 minutes spaced FX returns, and observed that the conditional variances share the same long–memory component, while the conditional means have a common antipersistent component.

The game of estimating and comparing the degrees of long–memory might be endless as one can always consider other financial assets and estimators. However, such an exercise would be of little interest, and a more interesting direction is to find the cause of these striking regularities in the long–memory properties, which are presumably the outcome of some common economic phenomenon.

3 Semiparametric and nonparametric tests and estimators

As we are investigating the properties of behavioral microeconomic models which cannot be reduced to a standard stochastic process, we consider here model independent tests and estimators, i.e., a class of tests and estimators which do not require a specific functional form or a particular distributional assumption on the stochastic process generating the data. We also analyze the behaviour and relative performance of these tests and estimators with our non-standard data generating process. A similar methodology has been followed in Kirman and Teyssière (2001, 2002) for analyzing other statistical properties of the behavioral models studied here.

We first consider a family of estimators and tests which are based on the approximation of the spectrum of an $I(d)$ process in a neighborhood of the zero frequency, as given in equation (2.2). Robinson (1994c, 1995) proposed two semiparametric estimators which are asymptotically normally distributed and robust to conditional heteroskedasticity of general form, including long–memory conditional heteroskedasticity, with the same limiting distribution as in the homoskedastic *i.i.d.* case.[11] We consider here only the second estimator as it is more efficient, its asymptotic distribution is independent of d, and a feasible optimal bandwidth for this estimator has been derived by Henry and Robinson (1996). Künsch (1987) suggested estimating the parameter d by replacing the analytical expression of the spectrum in the Whittle approximate Gaussian maximum likelihood estimator by its approximation given by equation (2.2). Robinson (1995) developed this idea and established the asymptotic properties of this estimator, obtained by solving the minimization problem:

$$\{\hat{C}, \hat{d}\} = \arg\min_{C,d} L(C,d) = \frac{1}{m}\sum_{j=1}^{m}\left\{\ln\left(C\lambda_j^{-2d}\right) + \frac{I(\lambda_j)}{C\lambda_j^{-2d}}\right\} \tag{3.1}$$

[10] These series are in ϑ-time, i.e., the intra-day seasonal component has been removed (See Dacorogna *et al.*, 1993). Since these series do not have the same time scale, we cannot evaluate the co-volatilities.

[11] The robustness properties have been established by Henry (2001b) and Robinson and Henry (1999) respectively.

where $I(\lambda_j)$ is the periodogram. We assume that the approximation (2.2) holds for a degenerate range of m Fourier frequencies $\lambda_j = 2\pi j/n$, $j = 1, \ldots, m \ll [n/2]$, where $[\cdot]$ denotes the integer part operator, bounded by the bandwidth parameter m, which increases with the sample size n but more slowly as $1/m + m/n \to 0$ as $n \to \infty$. Under appropriate conditions, which include the differentiability of the spectrum near the zero frequency and the existence of a moving average representation, the asymptotic distribution of this Gaussian estimator is

$$\sqrt{m}(\hat{d} - d) \sim N\left(0, \frac{1}{4}\right) \tag{3.2}$$

We use here the formula for the data driven optimal bandwidth proposed by Henry and Robinson (1996) which, as demonstrated by Henry (2001a), is robust to conditional heteroskedasticity of general form including the long–memory case.

We consider in this paper several model independent tests for $I(0)$ against fractionally integrated alternatives $I(d)$. Lobato and Robinson's (1998) nonparametric Lagrange multiplier test for $I(0)$ against $I(d)$ alternatives is also based on the approximation (2.2) of the spectrum of a long–memory process. In the univariate case, the t statistic is defined as:

$$t = \sqrt{m}\hat{C}_1/\hat{C}_0 \quad \text{with} \quad \hat{C}_k = m^{-1}\sum_{j=1}^{m} \nu_j^k I(\lambda_j) \quad \text{and} \quad \nu_j = \ln(j) - \frac{1}{m}\sum_{i=1}^{m}\ln(i) \tag{3.3}$$

where m is a bandwidth parameter. Under the null hypothesis of a $I(0)$ time series, the statistic t^2 is $\chi^2(1)$ distributed.

The three other tests for $I(0)$ against $I(d)$, are based on the assumption that under the null hypothesis of $I(0)$, the standardized series of the partial sums of the process $S_k = \sum_{j=1}^{k}\left(Y_j - \bar{Y}_n\right)$ satisfies a functional central limit theorem. What is required is only the existence of an autocorrelation consistent estimator of the variance. All these tests make use of the Newey and West (1987) heteroskedastic and autocorrelation consistent (HAC) variance estimator.

$$\hat{\sigma}^2(q) = \hat{\gamma}_0 + 2\sum_{i=1}^{q}\omega_i(q)\hat{\gamma}_i \quad \text{with} \quad \omega_i(q) \equiv 1 - \frac{1}{q+1} \tag{3.4}$$

where the sample auto-covariances $\hat{\gamma}_i$ at lag i account for the possible short-range dependence up to the q^{th} order. The main problem with the statistics considered here is the lack of statistical criteria for choosing the truncation order q although it should logically be related to the degree of autocorrelation and the length of the series.

Lo (1991) modified Hurst's (1951) R/S statistic, based on the range of the partial sum process S_k, by replacing the standard variance estimator by the HAC estimator. This new statistic is robust to short-range dependence and is defined as:

$$R/S(q) = \frac{1}{\hat{\sigma}(q)}\left[\max_{1\le k\le n} S_k - \min_{1\le k\le n} S_k\right] \tag{3.5}$$

If $q = 0$, Lo's statistic reduces to Hurst's R/S statistic. Under the null hypothesis of no long–memory, the statistic $n^{-\frac{1}{2}}R/S$ converges to a distribution equal to the range of a Brownian bridge on the unit interval: $\max_{0\le t\le 1} W^0(t) - \min_{0\le t\le 1} W^0(t)$ where W_t^0 is the Brownian bridge defined as $W^0(t) = W(t) - tW(1)$, $W(t)$ being the standardized Wiener process. Teverovsky, Taqqu and Willinger (1999) observed that the probability of accepting the null hypothesis of no long-range dependence depends significantly on q, and is over-estimated whatever the long–memory properties of the data. These authors suggest using this statistic together with other tests and estimators for long–memory analysis.[12]

The KPSS test, proposed by Kwiatkowski *et al.* (1992) and Lee and Schmidt (1996), based on the second moment of the process S_k, is defined as:

$$KPSS(q) = \frac{1}{n^2 \hat{\sigma}^2(q)} \sum_{k=1}^{n} S_k^2 \tag{3.6}$$

Giraitis *et al.* (2001) have shown that under the null hypothesis of $I(0)$, this statistic asymptotically converges to a well defined random variable $U = \int_0^1 \left(W^0(t)\right)^2 dt$, where $W^0(t)$ is a Brownian bridge. Furthermore, these authors pointed out that the cumulative distribution function of this statistic has a series expansion, involving parabolic cylinder functions, which converges very quickly.

Giraitis, Kokoszka, Leipus and Teyssière (2001) proposed a centering of the KPSS statistic. This statistic, denoted by V/S, is then based on the variance of the process S_k:

$$V/S(q) = \frac{1}{n^2 \hat{\sigma}^2(q)} \left[\sum_{k=1}^{n} S_k^2 - \frac{1}{n} \left(\sum_{k=1}^{n} S_k \right)^2 \right] = n^{-1} \frac{\hat{V}(S_1, \dots, S_n)}{\hat{\sigma}^2(q)} \tag{3.7}$$

The limiting distribution of this statistic is a well defined random variable $V = \int_0^1 \left(W^0(t)\right)^2 dt - \left(\int_0^1 W^0(t) dt\right)^2$ the distribution of which is linked to the distribution of the Kolmogorov statistic by a single change of variable. This statistic has uniformly higher power than the KPSS statistic, and is less sensitive to the choice of the order q than Lo's statistic.[13] Giraitis *et al.* (2001) have also shown that this statistic can be used in the detection of long–memory in volatility for the class of ARCH(∞) processes.

We can estimate semi-parametrically the degree of long–memory in volatility in the framework of the long–memory linear ARCH model developed by Giraitis, Robinson and Surgailis (2000), defined as:

$$R_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim D(0,1) \quad \text{with} \quad \sigma_t = \alpha + \sum_{j=1}^{\infty} \psi_j R_{t-j} \tag{3.8}$$

where the infinite sequence of coefficients $\{\psi_j\}_{j=1}^{\infty}$ has the rate of decay $\psi_j = O(j^{d-1})$, with $d \in (0, 1/2)$. Giraitis, Robinson and Surgailis (2000) have shown that under the condition $L(E\varepsilon_0^4)^{1/2} \sum_{j=1}^{\infty} \psi_j^2 < 1$, where $L = 7$ for the Gaussian case and $L = 11$ in other cases, there exist a stationary solution to the two previous equations, such that the sequence of squares $\{R_t^2\}_{t=1}^{\infty}$ has a covariance function the rate of decay of which is given by equation (2.1).

Giraitis, Kokoszka, Leipus and Teyssière (2000) have proposed several semi-parametric estimators for the degree of long–memory of the linear long–memory ARCH process, from its sequence of squares $\{R_t^2\}_{t=1}^{n}$. One of them is the standard R/S "pox-plot" analysis advocated by Mandelbrot and Wallis (1969), Mandelbrot and Taqqu (1979), among others. The two others estimators are new, and simply extend the "pox-plot" analysis to the KPSS and V/S statistics. Interested readers are referred to Beran (1994) and Giraitis *et al.* (2000) for the technical details of the implementation of the "pox-plot methods".

[12]When q increases, the power of this statistic tends quickly to its size. The conclusions of Teverovsky *et al.* (1999) should be qualified by the fact that these authors consider excessive values for q, e.g., 50.

[13]See Giraitis *et al.* (2001), Kirman and Teyssière (2002).

4 Microeconomic models for long–memory in the volatility

The microeconomic models used here are derived from Kirman (1991, 1993). Their basic foundation is the existence of two groups of agents, called chartists and fundamentalists, who differ by the rule which they use to forecast prices. One rule is based on economic fundamentals, and those who follow it are called "fundamentalists", and the other is based on extrapolation and is used by "chartists". The important feature of these models is that individuals change from being fundamentalists and become chartists and vice-versa. Thus, the groups are not fixed in size and this has consequences for market behaviour.

It is important to note that the models here, although of a sequential nature, are still equilibrium models. In other words, at each point of time the level of exchange rate is such that the total volume demanded is equal to the total volume supplied. On a daily basis, this may not be unreasonable but if we take price data at shorter intervals this sort of model becomes much less realistic. As soon as we look at prices at very high frequencies we are obliged to ask what those prices represent. This will depend on the type of market organisation but will at best reflect the terms at which transactions are made and not at all short term equilibrium. In an electronic trading system if the best offer of a particular share is 500 at \$100 it may be faced with a demand for 2000 at that price. Nevertheless 500 of the demand will be served and that price will be recorded even though it can not reasonably be considered as an equilibrium price. Thus, to model prices of this sort by an equilibrium process, even though this is common practice, is unrealistic. In particular, the foreign exchange market involves "market makers" trading independently and quoting "bids" and "asks". The published rates reflect some average of these.[14]

Why is this important? Simply because the sort of model here does not capture the statistical features of high-frequency data and for the reasons just given this is not surprising.

If the markets are efficient, the expected price $E(P_{t+1})$ of an asset at time $t+1$ conditional on the information set I_t is given by:

$$E(P_{t+1}|I_t) = P_t \tag{4.1}$$

In our setting, agents do not consider markets to be efficient and then assume that they can predict the next price level, i.e.,[15]

$$E(P_{t+1}|I_t) = \Delta P_{t+1}|I_t + P_t \tag{4.2}$$

where $\Delta P_{t+1}|I_t$ is the predicted price change at time $t+1$, given the information set I_t. Let P_t be the exchange rate at time t, chartists make the assumption that the exchange rate in the next period is a convex linear function of the previous prices, i.e.,

$$E^c\left(P_{t+1}|I_t\right) = \sum_{j=0}^{M_c} h_j P_{t-j} \quad \text{with} \quad \sum_{j=0}^{M_c} h_j = 1 \tag{4.3}$$

where h_j, $j = 0, \ldots, M_c$ are constants, M_c is the memory of the chartists. On the other hand, fundamentalists forecast the next price as:

$$E^f\left(P_{t+1}|I_t\right) = \bar{P}_t + \sum_{j=1}^{M_f} \nu_j (P_{t-j+1} - \bar{P}_{t-j}) \tag{4.4}$$

[14] For details of the mechanics of foreign exchange markets see e.g., Grabbe (1992).

[15] The basic features of the model would not be changed if we allowed for some more sophisticated extrapolary processes.

where ν_j, $j = 1, \ldots, M_f$ are positive constants, representing the degree of reversion to the fundamentals, M_f is the memory of the fundamentalists. This series of 'fundamentals' can be thought as the price if it were only to be explained by a set of relevant exogenous variables. We assume that the fundamentals $\bar{P}_t$ follow a random walk:

$$\bar{P}_t = \bar{P}_{t-1} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{4.5}$$

Individuals i, $i = 1, \ldots, N$ have a utility function given by:

$$U(W_{t+1}^i) = E(W_{t+1}^i) - \mu V(W_{t+1}^i) \tag{4.6}$$

where μ denotes the risk aversion coefficient, $E(.)$ and $V(.)$ denote the expectation and variance operators. Agents have the possibility of investing at home in a risk free asset or investing abroad in a risky asset. This foreign investment involves two risks, that of an exchange rate change, and that intrinsic in the foreign asset.

Denote by ρ_t the foreign interest rate, d_t^i the demand by the i^{th} individual for foreign currency, and r the domestic interest rate. The exchange rate P_t and the foreign interest rate ρ_t are considered by agents as independent random variables, with

$$\rho_t \sim N(\rho, \sigma_\rho^2) \quad \text{with} \quad \rho_t > r \tag{4.7}$$

Hence, the cumulated wealth of individual i at time $t+1$, W_{t+1}^i is given by:

$$W_{t+1}^i = (1 + \rho_{t+1}) P_{t+1} d_t^i + (W_t^i - P_t d_t^i)(1 + r) \tag{4.8}$$

Thus, we have:

$$E(W_{t+1}^i | I_t) = (1 + \rho) E^i(P_{t+1} | I_t) d_t^i + (W_t^i - P_t d_t^i)(1 + r) \tag{4.9}$$

$$V(W_{t+1}^i | I_t) = (d_t^i)^2 \zeta_t \quad \text{where} \quad \zeta_t = V\left(P_{t+1}(1 + \rho_{t+1})\right) \tag{4.10}$$

Demand d_t^i is found by maximising utility and writing the first order condition

$$(1 + \rho) E^i(P_{t+1} | I_t) - (1 + r) P_t - 2\zeta_t \mu d_t^i = 0 \tag{4.11}$$

where $E^i(.|I_t)$ denotes the expectation of an agent of type i. Let k_t be the proportion of fundamentalists at time t, the market demand is:

$$d_t = \frac{(1 + \rho)\left(k_t E^f(P_{t+1} | I_t) + (1 - k_t) E^c(P_{t+1} | I_t)\right) - (1 + r) P_t}{2\zeta_t \mu} \tag{4.12}$$

Now consider the exogenous supply of foreign exchange X_t, then the market is in equilibrium if aggregate supply is equal to aggregate demand, i.e., $X_t = d_t$, which gives

$$(1 + r) P_t = (1 + \rho)\left(k_t E^f(P_{t+1} | I_t) + (1 - k_t) E^c(P_{t+1} | I_t)\right) - 2\zeta_t \mu X_t \tag{4.13}$$

We assume that $2\zeta_t \mu X_t / (1 + \rho) = \gamma \bar{P}_t$. If $M_f = M_c = 1$, then the equilibrium price is given by

$$P_t = \frac{k_t - \gamma}{A} \bar{P}_t - \frac{k_t \nu_1}{A} \bar{P}_{t-1} + \frac{(1 - k_t) h_1}{A} P_{t-1} \tag{4.14}$$

with

$$A = \frac{1+r}{1+\rho} - (1-k_t)h_0 - k_t\nu_1 \tag{4.15}$$

Thus, for our so called "Havana–India" model, the foreign exchange rate P_t is a combination with varying coefficients of the previous price P_{t-1} and the fundamentals $\bar{P}_t$ and $\bar{P}_{t-1}$.

Since this paper has been motivated by the similarity of the long memory component of the volatilities and co-volatilities of FX rates, we wish to check whether our model is able to generate this empirical characteristic. We then consider a second exchange rate, denoted by P_t^*, which depends on a series of fundamentals $\bar{P}_t^*$, a foreign interest rate ρ_t^*, and two forecasts functions with parameters ν_1^*, h_0^* and h_1^*. We impose a common restriction on the two processes P_t and P_t^* by assuming that the proportion of fundamentalists k_t is the same for both processes. This assumption is reasonable as being a fundamentalist means relying on fundamentals, whatever they are, provided that there is no good reason for not relying on a particular series of fundamentals.

Thus, the process P_t^* is defined as

$$P_t^* = \frac{k_t - \gamma^*}{A^*}\bar{P}_t^* - \frac{k_t\nu_1^*}{A^*}\bar{P}_{t-1}^* + \frac{(1-k_t)h_1^*}{A^*}P_{t-1}^* \tag{4.16}$$

where

$$A^* = \frac{1+r}{1+\rho^*} - (1-k_t)h_0^* - k_t\nu_1^* \tag{4.17}$$

The last building block of the model consists in introducing the process governing the evolution of k_t, i.e., the proportion of agents making a forecast based on fundamentals in the whole population. It is assumed that:

1. Agents interact,
2. Agents communicate their beliefs on the next period forecast through a particular epidemiologic process introduced by Föllmer.

Since the parameters of the epidemiologic model are independent of the previous parameters of the model, the proportion of fundamentalists and the forecasts of agents is independent of the economic variables.

Let N be the total number of agents and ϑ_t be the number of agents with a fundamentalist forecast at time t. We assume that pairs of agents meet at random and that the probability that the first agent is converted to the opinion of the second one is equal to $(1-\delta)$. Furthermore, each agent can independently change his opinion with probability ξ. This probability insures that the process is not trapped in the extremes, i.e., all agents are chartists or all agents are fundamentalists.

Given that the state of the process is summarized by the value of ϑ_t, its evolution is defined by the following transition matrix:[16]

$$\Pr(\vartheta, \vartheta+1) = \left(1 - \frac{\vartheta}{N}\right)\left(\xi + (1-\delta)\frac{\vartheta}{N-1}\right) \tag{4.18}$$

$$\Pr(\vartheta, \vartheta-1) = \frac{\vartheta}{N}\left(\xi + (1-\delta)\frac{N-\vartheta}{N-1}\right) \tag{4.19}$$

$$\Pr(\vartheta, \vartheta) = 1 - \Pr(\vartheta, \vartheta+1) - \Pr(\vartheta, \vartheta-1) \tag{4.20}$$

[16]The limit distribution of the Markov chain defined by this transition matrix is a Beta distribution. This result proved by Föllmer is reproduced in Kirman (1991).

After the meetings, the proportion of fundamentalists is equal to ϑ_t/N. However, agents observe this proportion with error, i.e., agent i observe $k_{i,t}$, with:

$$k_{i,t} = \frac{\vartheta_t}{N} + \varepsilon_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \sim N(0, \sigma_\vartheta^2) \tag{4.21}$$

If agent i observe that $k_{i,t} \geq 0.5$, then he will make a fundamentalist forecast, otherwise he will make a chartist forecast. Thus, the proportion k_t of agents making a fundamentalist forecast is given by:

$$k_t = N^{-1} \sum_{i=1}^{N} \mathbf{1}_{(k_{i,t} \geq 0.5)} \tag{4.22}$$

where $\mathbf{1}_{(cond)}$ denotes the indicator function, which is equal to 1 if *cond* is true, and 0 otherwise.

The herding behaviour of the process k_t on the extremes depends on ξ and σ_ϑ. If ξ becomes smaller, then the process k_t will spend more time on the extremes 0 and 1. The parameter σ_ϑ measures the accuracy of observation of the proportion of fundamentalists; see equation (4.21). If σ_ϑ becomes smaller, the prevailing opinion is observed with more accuracy, which results in massive swings of opinion. There are two related reasons for such behaviour. Firstly, Keynes pointed out that individuals trades are concerned about what "market opinion" is rather than about fundamental values. This, he argued, is because it is less risky to be wrong with the crowd rather than being wrong alone. Alternatively on could think of a Nash equilibrium in which if all people have the same opinion, no one has an incentive to deviate. As we will see in the next section, these two parameters govern the level of long-range dependence in the volatility of the simulated returns.

5 Monte Carlo analysis of the process

5.1 Characteristics of the simulations

We generate 10000 samples of 3000 observations, which is the typical size for daily financial time series. We adjust the parameters of the model so that the series of returns $R_t = \ln(P_t/P_{t-1})$ generated by the epidemiologic model is $I(0)$, and the volatility of the series of simulated returns display long-range dependence, i.e., are $I(d)$ with $d \in (0, 1/2)$. We choose as proxy of the volatility the series of absolute returns $|R_t|$ and squared returns R_t^2.

We detect the presence of long–memory by using the tests for $I(0)$ against $I(d)$ alternatives presented in section **3**. For the test based on the partial sum process, i.e., Lo's test, the KPSS test, and the V/S test, we consider the truncation orders $q = 0, 1, 2, \ldots, 5, 10, 15, \ldots, 30$ for the HAC variance estimator. For the Lobato and Robinson (1998) test, we use a grid of bandwidths $m = 60, 84, 108, 132, 156$ as in Lobato and Savin (1998). We tune the parameters of the model so that the degree of long–memory, estimated with Robinson's (1995) estimator and the optimal bandwidth m_{opt} by Henry and Robinson (1996), is the same as the one observed in financial time series. For this Gaussian estimator, we use the optimal bandwidth m_{opt}, and the grid $m = 84, 108, 132, 156$.

We choose the following values for the parameters of the model:

- Number of agents $N = 1000$,
- At time $t = 0$, the number of fundamentalists is equal to the number of chartists, i.e., $k_0 = 0.5$,

- $P_0 = 1000$, initial value of the exchange rate,
- $\bar{P}_0 = 1050$, initial value of the fundamentals series,
- $\sigma_\varepsilon^2 = 10.0$,
- Annual foreign interest rate $\rho = 0.07$, the daily foreign interest rate is then equal to 0.00018538,
- Annual domestic interest rate $r = 0.04$, thus the daily domestic interest rate is equal to 0.000133668,
- $\nu_1 = 0.59$, $h_0 = 0.625$, $h_1 = 1 - h_0$,
- $\delta = 0.010$,
- $\xi = 0.000325$;
- $\sigma_\vartheta^2 = 0.33$.

For the bivariate process (P_t, P_t^*), the parameters of the process P_t remain the same, while the parameters of the process P_t^* are:

- $\rho^* = 0.08\%$, at an annual rate, the daily rate is then equal to 0.00021087,
- $P_0^* = 1500$, initial value of the second series of exchange rate,
- $\bar{P}_0^* = 1550$, initial value of the second series of fundamentals, $\sigma_\varepsilon^2 = 15.0$,
- $\nu_1^* = 0.58$, $h_0^* = 0.625$, $h_1^* = 1 - h_0^*$.

5.2 Simulation results

The series of simulated returns match the empirical properties of the two first moments of absolute returns. Unlike standard long–memory processes, the series of simulated absolute returns are not trended. Figures 6 and 7 display two series of simulated absolute returns, which are similar to what is empirically observed, see Figures 4 and 5. For Figures 6 and 7 the estimated values of d, obtained with Robinson's (1995) Gaussian estimator and the optimal bandwidth, are respectively $\hat{d} = 0.4425$ and $\hat{d} = 0.3914$. A standard long–memory process with such degrees of dependence will have a marked trend.

However, the series of simulated absolute returns display strong dependence: Figure 3 displays the ACF of a series of absolute simulated returns which resembles to the empirical ACF of absolute returns displayed in Figures 1 and 2: there is a significant autocorrelation as observed with real data.

Simulation results reported in tables 6 and 7 show that the four statistical tests accept 95% of the time the null hypothesis of no long-range dependence for the differenced series $(1 - L)P_t$.

Table 4 display the mean of the degree of long–memory estimated with Robinson's (1995) Gaussian estimator. We observe that the Monte Carlo standard deviations of the Gaussian estimator are greater than the theoretical standard deviations $(2\sqrt{m})^{-1}$ obtained under some assumptions on the process generating the data:[17] for our sample size, the theoretical standard

[17] See Robinson (1995).

deviations for the bandwidths $84, 108, 132, 156$ are equal to 0.0545, 0.0481, 0.0435 and 0.0400 respectively. This means that our microeconomic model based data generating process, henceforth DGP, is more complex than a standard linear long–memory process.

The use of a stochastic process for modelling the evolution of the process k_t allows us to control the link between herding behaviour and long–memory. This makes the analysis easier than in Kirman (1999) and Gaunersdorfer and Hommes (2000) who used an evolutionary mechanism based on the relative performance of the forecast functions of chartists and fundamentalists. We consider several values for the parameters ξ, which is the probability that an agent independently changes his opinion, and the parameter σ^2_ϑ which represents the accuracy of observation of the proportion of fundamentalists, see equation (4.21). When $\sigma^2_\vartheta = 0.33$ and $\xi = 0.000325$, the estimated degree of long–memory d in the absolute returns is equal to 0.35, while when $\sigma^2_\vartheta = 0.60$ and $\xi = 0.000325$, the estimated value of d decreases to 0.30, and when $\xi = 0.01$ and $\sigma^2_\vartheta = 0.33$, this degree decreases to 0.20. Thus, the estimated degree of long-range dependence of the model is linked to the herding behaviour of agents.

Table 5 p 20 reports the estimated values of d from the three "pox-plot" based semiparametric estimators, which do not differ too much from the results obtained from the local Whittle estimator.[18] We can conclude that our DGP differs from the long–memory linear ARCH model by Giraitis, Robinson and Surgailis (2000), which is not surprising.

Results on tables 6 and 7, show that the Lobato-Robinson's test, Lo's test, the KPSS test and the V/S test detect the presence of long–memory in the simulated series of absolute and squared returns. The Lobato–Robinson test, the V/S test and Lo's test have the same power and have more power than the KPSS test. Giraitis *et al.* (2001) conjectured that this higher power is due to the smaller variance of the V/S statistic.

When interpreting these results, we have to keep in mind that the performance of these asymptotic tests has been originally assessed with respect to a standard DGP. Given that we apply them to a non-standard DGP, we have to adjust them on the correct size basis. Davidson and MacKinnon (1998) suggest to draw the size-power tradeoff curve, which graphs the probability of rejecting the null hypothesis when it is false against the probability of rejecting the null hypothesis when it is true. Figures 10 and 11 display the size-power curves which are obtained by plotting the empirical distribution function, henceforth EDF, $\tilde{F}(x)$ of the P-values of the DGP which does not satisfy the null hypothesis against the EDF $\hat{F}(x)$ of the P-values of a DGP which satisfies this null hypothesis. If the test is pivotal, i.e., its distribution is independent of any unknown feature of the process generating the data, the choice of the DGP satisfying the null hypothesis is not important. However, if the test is not pivotal, Davidson and MacKinnon (1996) suggest to choose the "pseudo-null" DGP, which is in the set of DGP satisfying the null hypothesis, the closest, according to the Kullback-Leibler criterion, to the DGP which does not satisfy the null hypothesis.

For $q = 0$, the size–power curves for all the statistics are not distinguishable, thus we display the curves for $q = 30$. The size-power curves of Lo's statistic and the V/S test are over the curve of the KPSS statistic. For small samples, e.g., 500 observations, the size-power curve of the V/S test is over the two other curves, see Kirman and Teyssière (2002). The V/S statistic is slightly less sensitive to the choice of q than Lo's statistic.

Finally, table 8 p 21 gives the simulation results for the bivariate process (R_t, R^*_t), with $R_t = \ln(P_t/P_{t-1})$ and $R^*_t = \ln(P^*_t/P^*_{t-1})$. These results show that the estimated values of d are

[18] Interestingly, the Monte Carlo standard errors slightly differ from the ones of the long–memory linear ARCH process. See Giraitis *et al.* (2000).

close for the series of volatilities $|R_t|$, $|R_t^*|$ and co-volatilities $\sqrt{|R_t, R_t^*|}$ for all the values of the bandwidth parameter m. Our model is then able to generate the empirical property of common long-range dependence in the volatilities and co-volatilities of FX rates.

Figures 8 and 9 display two examples of the evolution of the process k_t. The process never herds on the extremes 0 and 1, which is consistent with what is empirically observed on financial markets: agents never all swing together to one belief. There is a tendency for one view to predominate at any point in time, but there is always a minority with the other view and in time this minority takes over.

6 Conclusion

We have presented here a new class of models of financial markets based on the idea that individual agents interact stochastically. These models replicate the empirically observed characteristics of daily exchange rates series. These characteristics are that the returns are uncorrelated, while the absolute returns and squared returns display long–memory.

We have explained the nature of "long–memory" and show how it may be detected in the sort of non-standard data generating process yielded by our models. The fact that the process is non-standard means that we had to use nonparametric and semiparametric methods.

While Evans (1991) pointed out that some bubble like features which were deliberately introduced into time series would not be detected by standard procedures, he did not provide an economic model which would generate this structure.

Our models generate "herding behaviour" and swings of opinion which give rise to bubble like features, see Kirman and Teyssière (2001, 2002). However, the important feature of these models, from the point of view of this paper, is that they also generate the sort of long–memory which can be detected and is actually present in empirical series.

What is most interesting is that this sort of long–memory seems to be intimately linked to the tendency of the markets participants to herd on the extremes. Thus "bubbles" seems to be linked to long–memory through this herding behaviour.

References

Aoki, M. (1996): *A New Approach to Macroeconomic Modeling.* Cambridge University Press: New York.

Baillie, R. T., Bollerslev, T. and H. O. Mikkelsen (1996): "Fractionally integrated generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, 74, 3–30.

Banerjee, A. (1992): "A simple model of herd behavior," *Quarterly Journal of Economics*, 107, 797–818.

Beran, J. (1994): *Statistics for Long–memory Processes.* Chapman and Hall.

Bikhchandani, S., Hirshleifer, D. and I. Welch (1992): "A theory of fads, fashion, custom and cultural changes as informational cascades," *Journal of Political Economy*, 100, 992–1027.

Blanchard, O. and M.W. Watson (1982): "Bubbles, rational expectations and financial markets," in P. Wachtel ed., *Crisis in the Economic and Financial System*, Lexington, MA: Lexington Books, 295–315.

Box-Steffenmaier, J.M. and R.M. Smith (1996): "The dynamics of aggregate partisanship," *American Political Science Review*, 90, 567–580.

Breidt, F.J., Crato, N. and P. de Lima (1998): "On the detection and estimation of long–memory in stochastic volatility," *Journal of Econometrics*, 83, 324–352.

Byers, D., Davidson, J., and D. Peel (1997): "Modelling political popularity: an analysis of long range dependence in opinion poll series," *Journal of the Royal Statistical Society*, A 160, 471–490.

Dacorogna, M.M., Müller, U.A., Nagler, R.J., Olsen, R.B. and O.V. Pictet (1993): "A geographical model for the daily and weekly seasonal volatility in the foreign exchange market," *Journal of International Money and Finance*, 12, 413–438.

Dacorogna, M.M., Müller, U.A., Pictet, O.V. and C.G. De Vries (1995): "The distribution of extremal foreign exchange rate returns in extremely large data sets," *Olsen & Associates Research Paper* UAM 1992-10-22.

Davidson, R. and J.G. MacKinnon (1996): "The power of bootstrap tests," *Queen's Institute for Economic Research Discussion Paper* 937.

Davidson, R. and J.G. MacKinnon (1998): "Graphical methods for investigating the size and power of hypothesis tests," *The Manchester School*, 66, 1–26.

Day, R. and B. Huang (1990): "Bulls, bears and market sheep," *Journal of Economic Behavior and Organization*, 14, 299–329.

Dickey, D.A. and W.A. Fuller (1979): Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association*, 74, 427–431.

Ding, Z. and C.W.J. Granger (1996): "Modelling volatility persistence of speculative returns. A new approach," *Journal of Econometrics*, 73, 185–215

Durlauf, S.N. (1997): "Statistical mechanics approaches to socioeconomic behaviour," in W.B. Arthur, S.N. Durlauf and D.A. Lane eds., *The Economy as an Evolving Complex System*, Addison Wesley: Reading, Massachussets, 81–104.

Evans, G.W. (1991): "Pitfalls in testing for explosive bubbles in asset prices," *American Economic Review*, 81, 922–930.

Gallant, A.R., Hsu, C-T. and G. Tauchen (1998): "Calibrating volatility diffusions and extracting integrated volatility," in *Proceedings of the Second High Frequency Data in Finance Conference, HFDF-II,* , Olsen & Associates, 1-3 April 1998, Zurich, Switzerland.

Gaunersdorfer, A. and C. H. Hommes (2000): "A nonlinear structural model for volatility clustering," *CeNDEF Working Paper* 00-02.

Giraitis, L. and D. Surgailis (2001): "ARCH–type bilinear models with double long–memory", *Preprint.*

Giraitis, L., Kokoszka, P.S. and R. Leipus (2000): "Stationary ARCH models: dependence structure and central limit theorem," *Econometric Theory*, 16 , 3–22.

Giraitis, L., Kokoszka, P.S., Leipus R. and G. Teyssière (2001): "Rescaled variance and related tests for long–memory in volatility and levels," *Journal of Econometrics*, forthcoming.

Giraitis, L., Robinson, P.M. and D. Surgailis (2000): "A model for long–memory conditional heteroskedasticity," *Annals of Applied Probability*, 10, 1002–1024.

Giraitis, L., Kokoszka, P.S., Leipus, R. and G. Teyssière (2000): "Semiparametric estimation of the intensity of long–memory in conditional heteroskedasticity," *Statistical Inference for Stochastic Processes*, 3, 113–128.

Grabbe, J.O. (1992): *International Financial Markets.* Prentice Hall, New York.

Granger, C.W.J. (1980): "Long–memory relationships and the aggregation of dynamic models," *Journal of Econometrics*, 14, 227–238.

Granger, C.W.J. and R. Joyeux (1980): "An introduction to long–memory time series models and fractional differencing," *Journal of Time Series Analysis*, 1, 15–29.

Granger C.W.J. and Z. Ding (1995): "Some properties of absolute returns, an alternative measure of risk," *Annales d'Économie et de Statistique*, 40, 67–91.

Granger, C.W.J. and Z. Ding (1996): "Varieties of long–memory models," *Journal of Econometrics*, 73, 61–77.

Hamilton, J.D. (1989): "A new approach to the economic analysis of nonstationary time series and the business cycle," *Econometrica*, 57, 357–384.

Harvey, A.C. (1998): "Long–memory in stochastic volatility," in J. Knight and S. Satchell eds., *Forecasting Volatility in Financial Markets*, Oxford: Butterworth-Heineman, 307–320

Henry, M. and R. Payne (1997): "An investigation of the long range dependence in intra-day foreign exchange rate volatility," *LSE Financial Market Group Discussion Paper* 264.

Henry, M. (2001a): "Robust automatic bandwidth for long–memory," *Journal of Time Series Analysis*, 22, 293–316.

Henry, M. (2001b): "Averaged periodogram spectral estimation with long–memory conditional heteroskedasticity," *Journal of Time Series Analysis*, 22, 431–459.

Henry, M. and P.M. Robinson (1996): "Bandwidth choice in Gaussian semiparametric estimation of long range dependence," in P.M. Robinson and M. Rosenblatt eds., *Athens Conference on Applied Probability and Time Series Analysis, Time Series Analysis, In Memory of E.J. Hannan*, 2, LNS 115. Springer Verlag: New York, 220–232.

Hosking, J.R.M. (1981): "Fractional differencing," *Biometrika*, 68, 165–176.

Hurst, H.E. (1951): "Long-term storage capacity of reservoirs," *Transactions of the American Society of Civil Engineers*, 116, 770–799.

Kazakevičius, V. and R. Leipus (2002): "On stationarity in the ARCH(∞) model," *Econometric Theory*, **18**, 1–16.

Kazakevičius, V., Leipus, R. and M-C. Viano (2000): "Stability of random coefficients autoregressive conditionally heteroskedastic models," *Preprint.*

Kirman, A. (1991): "Epidemics of opinion and speculative bubbles in financial markets," in M. Taylor ed., *Money and Financial Markets*, chapter 17, Macmillan: London.

Kirman, A. (1993): "Ants, rationality and recruitment," *The Quarterly Journal of Economics*, 108, 137–156.

Kirman, A. (1999): "On the transitory nature of gurus," *Preprint.*

Kirman, A. and G. Teyssière (2001): "Testing for bubbles and change–point," *Preprint.*

Kirman, A. and G. Teyssière (2002): "Bubbles and long range dependence in asset prices volatilities," in C.H. Hommes, R. Ramer and C. Withagen eds., *Equilibrium, Markets and Dynamics.* Springer Verlag, forthcoming.

Künsch, H.R. (1987): "Statistical aspects of self-similar processes," in Yu. Prohorov and V.V. Sazanov eds., *Proceedings of the First World Congress of the Bernoulli Society*, 1, VNU Science Press: Utrecht, 67–74.

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Y. Shin (1992): "Testing the null hypothesis of stationarity against the alternative of a unit root: how sure are we that economic series have a unit root?" *Journal of Econometrics*, 54, 159–178.

Lee, D. and P. Schmidt (1996): "On the power of the KPSS test of stationarity against fractionally-integrated alternatives," *Journal of Econometrics*, 73, 285–302.

Lo, A.W. (1991): "Long-term memory in stock market prices," *Econometrica*, 59, 1279–1313.

Lobato, I. and P.M. Robinson (1998): "A nonparametric test for I(0)," *Review of Economic Studies*, 65, 475–495.

Lobato, I. and N.E. Savin (1998): "Real and spurious long–memory properties of stock market data," *Journal of Business and Economic Statistics*, 16, 261–283.

MacKinnon, J.G. (1994): "Approximate asymptotic distribution functions for unit-root and cointegration tests," *Journal of Business and Economic Statistics*, 12, 167–176.

MacKinnon, J.G. (1996): "Numerical distribution functions for unit-root and cointegration tests," *Journal of Applied Econometrics*, 11, 601–618.

Mandelbrot, B.B. (1963): "The variation of certain speculative prices," *Journal of Business*, 36, 384–419.

Mandelbrot, B.B. and J.M. Wallis (1969): "Computer experiments with fractional Gaussian noises," *Water Resources Research*, 5, 228–267.

Mandelbrot, B.B. and M.S. Taqqu (1979): "Robust R/S analysis of long-run serial correlation," in *42nd Session of the International Statistical Institute, Manila, Book* 2, 69–99.

Mandelbrot, B.B., Fisher, A. and L. Calvet (1997): "A multifractal model of asset returns," *Cowles Foundation Discussion Paper* 1164.

Newey, W.K. and K.D. West (1987): "A simple positive definite, heteroskedasticity and autocorrelation consistent covariance matrix," *Econometrica*, 55, 703–708.

Odaki, M. (1993): "On the invertibility of fractionally differenced ARIMA processes," *Biometrika*, 80, 703–709.

Ray, B.K. and R.S. Tsay (2000): "Common long-range dependence in daily stock volatilities," *Journal of Business and Economics Statistics*, 18, 254–262.

Robinson, P.M. (1991): "Testing for strong serial correlation and dynamic conditional heteroskedasticity in multiple regression," *Journal of Econometrics*, 47, 67–84.

Robinson, P.M. (1994a): "Time series with strong dependence," in C.A. Sims ed., *Advances in Econometrics, Sixth World Congress*, Cambridge University Press, 47–95.

Robinson, P.M. (1994b): "Efficient tests of nonstationary hypotheses," *Journal of the American Statistical Association*, 89, 1420–1437.

Robinson, P.M. (1994c): "Semiparametric analysis of long–memory time series," *The Annals of Statistics*, 22, 515–539.

Robinson, P.M. (1995): "Gaussian semiparametric estimation of long range dependence," *The Annals of Statistics*, 23, 1630–1661.

Robinson, P.M. (2001): "The memory of stochastic volatility models," *Journal of Econometrics*, 101, 195–218.

Robinson, P.M. and M. Henry (1999): "Long and short memory conditional heteroskedasticity in estimating the memory parameter in levels," *Econometric Theory*, 15, 299–336.

Sowell, F. (1990): "The fractional unit root distribution," *Econometrica*, 58, 495–505.

Taylor, S.J. (1986): *Modelling Financial Time Series.* Wiley.

Teverovsky, V., Taqqu, M.S. and W. Willinger (1999): "A critical look at Lo's modified R/S statistic," *Journal of Statistical Planning and Inference*, 80, 211–227.

Teyssière, G. (1997): "Modelling exchange rates volatility with multivariate long–memory ARCH processes," *GREQAM DT* 97B03. In revision for the *Journal of Business and Economic Statistics.*

Teyssière, G. (1998a): "Multivariate long–memory ARCH modelling for high frequency foreign exchange rates," in *Proceedings of the HFDF-II Conference*, Olsen and Associates, 1-3 April 1998, Zurich, Switzerland.

Teyssière, G. (1998b): "Nonlinear and semiparametric long–memory ARCH," *Preprint*.

Welch, I. (1992): "Sequential sales, learning and cascades," *Journal of Finance*, 47, 695–732.

Willinger, W., Paxson, V. and M.S. Taqqu (1998): "Self similarity and heavy tails: structural modeling of network traffic," in R.J. Adler, R.E. Feldman and M.S. Taqqu eds., *A Practical Guide To Heavy Tails: Statistical Techniques and Applications*, Birkhauser: Boston.

Woodford, M. (1990): "Learning to believe in sunspots," *Econometrica*, 58, 277–307.

A Empirical evidence

We report here the estimation results of the degree of long memory in the volatility by using Robinson's (1995) Gaussian estimator.

Table 1: Estimation of the fractional degree of integration for the series of absolute returns on Pound-US Dollar $|R_{1,t}|$, Deutschmark-US Dollar $|R_{2,t}|$, French Franc-US Dollar $|R_{3,t}|$, $\sqrt{|R_{1,t}R_{2,t}|}$, $\sqrt{|R_{1,t}R_{3,t}|}$, $\sqrt{|R_{2,t}R_{3,t}|}$ for the period January 1986 - January 1997.

m	$\|R_{1,t}\|$	$\|R_{2,t}\|$	$\|R_{3,t}\|$	$\sqrt{\|R_{1,t}R_{2,t}\|}$	$\sqrt{\|R_{1,t}R_{3,t}\|}$	$\sqrt{\|R_{2,t}R_{3,t}\|}$
$[n/4]$	0.2083	0.1682	0.1735	0.1847	0.2014	0.1745
$[n/8]$	0.2641	0.2911	0.2447	0.2777	0.2629	0.2512
$[n/16]$	0.3740	0.3767	0.3674	0.3879	0.3927	0.3682

Table 2: Estimation of the fractional degree of integration for the series of absolute returns on Pound-French Franc $|R_{1,t}|$, Deutschmark-French Franc $|R_{2,t}|$, $\sqrt{|R_{1,t}R_{2,t}|}$, for the period January 1986 - January 1997.

m	$\|R_{1,t}\|$	$\|R_{2,t}\|$	$\sqrt{\|R_{1,t}R_{2,t}\|}$
$[n/4]$	0.1661	0.1729	0.1448
$[n/8]$	0.2116	0.2167	0.1823
$[n/16]$	0.2690	0.2836	0.2489

Table 3: Estimation of the fractional degree of integration for the series of absolute returns on US Dollar-Deutschmark $|R_{DEM,t}|$, US Dollar-Yen $|R_{YEN,t}|$, US Dollar-Swiss Franc $|R_{CHF,t}|$, and US Dollar-Pound $|R_{GBP,t}|$.

m	$\|R_{DEM,t}\|$	$\|R_{YEN,t}\|$	$\|R_{CHF,t}\|$	$\|R_{GBP,t}\|$
$[n/4]$	0.1738	0.1776	0.1961	0.1810
$[n/8]$	0.2076	0.2252	0.2341	0.2138
$[n/16]$	0.2566	0.2450	0.2597	0.2550

B Monte Carlo results

Table 4: Gaussian estimates of d. (Monte Carlo S.E. in parenthesis.)

m	R_t	$\|R_t\|$	R_t^2
m_{opt}	0.0020 (0.0578)	0.3509 (0.0976)	0.3219 (0.0944)
84	-0.0063 (0.0712)	0.4806 (0.1205)	0.4483 (0.1186)
108	-0.0032 (0.0629)	0.4350 (0.1100)	0.4049 (0.1078)
132	-0.0005 (0.0580)	0.4009 (0.1021)	0.3723 (0.0996)
156	0.0013 (0.0550)	0.3742 (0.0962)	0.3468 (0.0931)

Table 5: "Pox–plot" estimates of d based on the squared returns series R_t^2. (Monte Carlo S.E. in parenthesis.)

	R/S estimate of d	V/S estimate of d	KPSS estimate of d
d	0.3086 (0.0872)	0.3259 (0.1043)	0.3870 (0.1172)

Table 6: Lobato–Robinson test. Test size 5%

m	$R_t : \Pr(d = 0)$	$\|R_t\| : \Pr(d > 0)$	$R_t^2 : \Pr(d > 0)$
60	0.9534	0.9940	0.9936
84	0.9444	0.9947	0.9946
108	0.9317	0.9956	0.9960
132	0.9194	0.9959	0.9955
156	0.9051	0.9963	0.9959

Table 7: Tests for long-range dependence on the series of differenced prices, absolute simulated returns $|R_t|$, and squared simulated returns R_t^2. Test size 5%.

	$R_t : \Pr(d=0)$			$|R_t| : \Pr(d>0)$			$R_t^2 : \Pr(d>0)$		
q	R/S	KPSS	V/S	R/S	KPSS	V/S	R/S	KPSS	V/S
0	0.9268	0.9494	0.9403	0.9955	0.9891	0.9937	0.9960	0.9886	0.9936
1	0.9336	0.9491	0.9442	0.9944	0.9865	0.9927	0.9946	0.9861	0.9922
2	0.9416	0.9482	0.9470	0.9934	0.9850	0.9917	0.9937	0.9847	0.9912
3	0.9470	0.9484	0.9495	0.9928	0.9844	0.9912	0.9931	0.9839	0.9908
4	0.9506	0.9485	0.9506	0.9926	0.9834	0.9903	0.9926	0.9829	0.9902
5	0.9530	0.9495	0.9530	0.9925	0.9825	0.9901	0.9918	0.9819	0.9899
10	0.9610	0.9501	0.9575	0.9913	0.9790	0.9893	0.9896	0.9778	0.9877
15	0.9618	0.9487	0.9595	0.9891	0.9752	0.9865	0.9860	0.9719	0.9855
20	0.9643	0.9491	0.9595	0.9850	0.9681	0.9837	0.9834	0.9663	0.9827
25	0.9657	0.9499	0.9591	0.9810	0.9617	0.9801	0.9784	0.9590	0.9790
30	0.9655	0.9499	0.9602	0.9767	0.9537	0.9762	0.9723	0.9508	0.9752

Table 8: Gaussian estimates of d for the bivariate series of simulated absolute returns $|R_t|$, $|R_t^*|$, $\sqrt{|R_t R_t^*|}$. (Monte Carlo S.E. in parenthesis.)

m	$|R_t|$	$|R_t^*|$	$\sqrt{|R_t R_t^*|}$
m_{opt}	0.3509 (0.0976)	0.3457 (0.0987)	0.3939 (0.1016)
84	0.4806 (0.1205)	0.4748 (0.1203)	0.5297 (0.1240)
108	0.4350 (0.1100)	0.4291 (0.1103)	0.4812 (0.1138)
132	0.4009 (0.1021)	0.3947 (0.1031)	0.4445 (0.1061)
156	0.3742 (0.0962)	0.3687 (0.0973)	0.4163 (0.1004)

C Graphs

Figure 1: Autocorrelation function of the absolute returns $|R_t|$ on Pound-Dollar

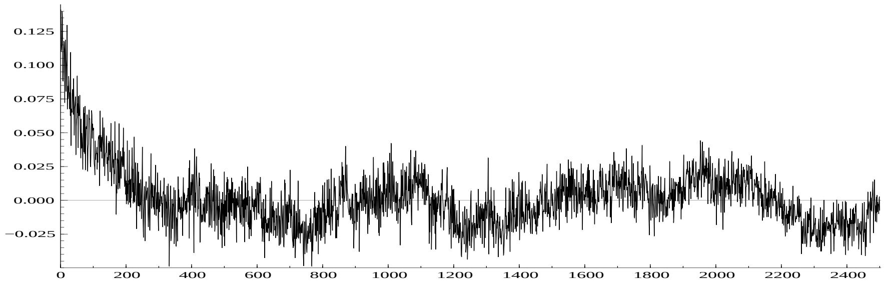


Figure 2: Comparison of ACF of $|R_t|$ (in grey) with the averaged ACF of 2000 simulations of a FINGARCH-t process (in black)

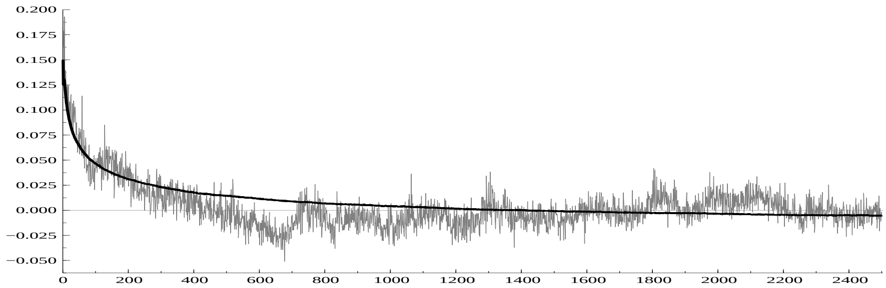


Figure 3: Autocorrelation function of the absolute returns $|R_t|$ of a simulation

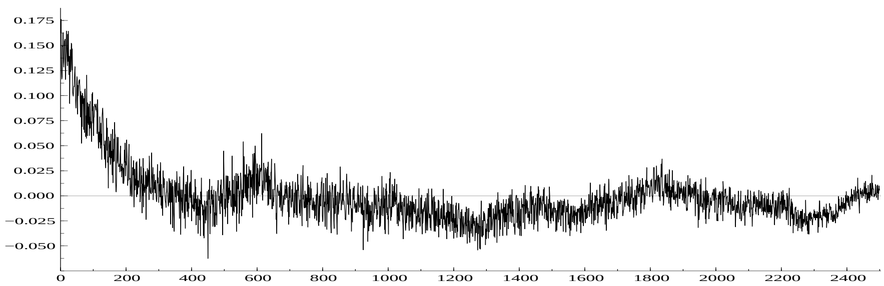


Figure 4: Series of absolute returns $|R_t|$ Deutschmark-Dollar

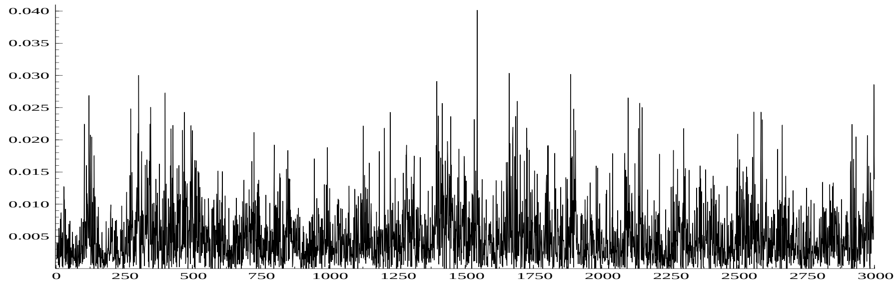


Figure 5: Series of absolute returns $|R_t|$ Pound-Dollar

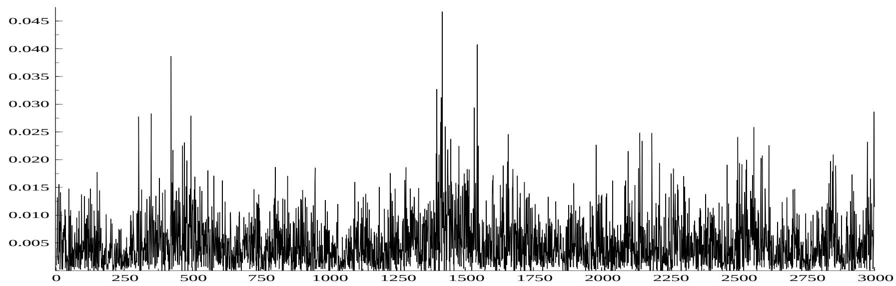


Figure 6: Series of simulated absolute returns $|R_t|$

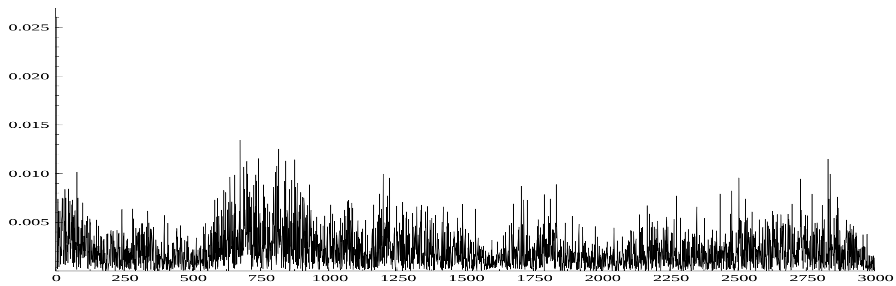


Figure 7: Series of simulated absolute returns $|R_t|$

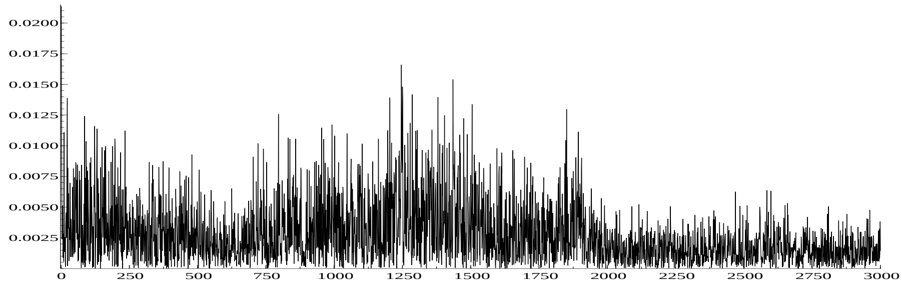


Figure 8: Evolution of the process k_t.

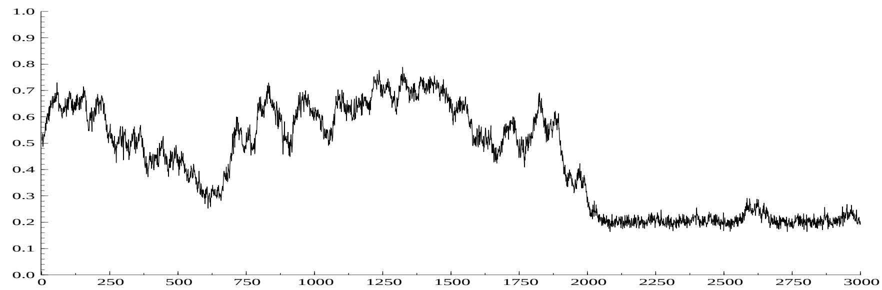
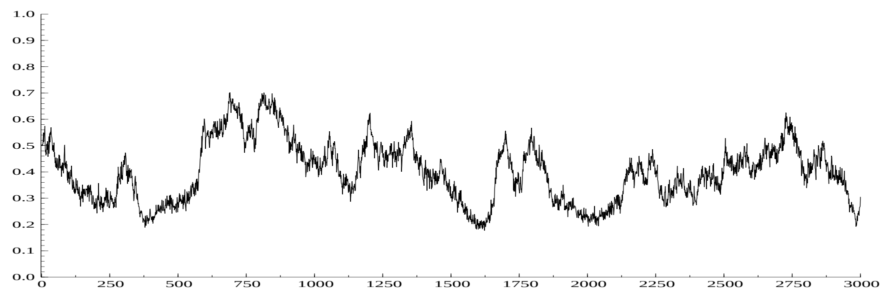


Figure 9: Evolution of the process k_t.



D Size-power curves

Figure 10: Power-size curves: absolute returns $|R_t|$

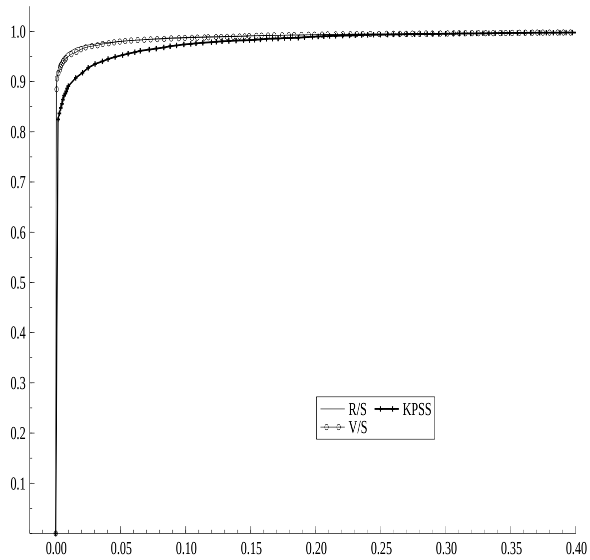


Figure 11: Power-size curves: squared returns R_t^2